Why Rent When You Can Buy?

A Theory of Repurchase Agreements*

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Abstract

In a model with bilateral trading, we explain why agents prefer to conduct repurchase agreements than asset sales. Absent bilateral trading frictions, repos have no role despite uncertainty about future valuations. With pairwise meetings, agents prefer to sell (or buy) assets whenever they face little uncertainty regarding the future use of the asset. As agents become more uncertain of the value of holding the asset, repos become more prevalent. Pairwise matching and search alone is sufficient to explain why repo markets exist and there is no need to introduce random matching, information asymmetries or other market frictions.

Keywords: Repo, over-the-counter market, bargaining, bilateral matching, directed search

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1. Introduction

Market participants – investment banks, brokers/dealers and market makers alike – borrow cash and financial securities, including sovereign and corporate bonds on the market for

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repurchase agreements (repos).[1] A repo is a spot sale combined with a forward contract: The lender sells the asset (at the spot price) with the agreement to buy it back from the borrower (at the forward price) at a later date. Therefore a repo looks like a loan with an interest payment that equals the difference between the forward price and the spot price. The crucial difference with a loan is that the buyer (the borrower of the asset) must have enough resources to buy the asset in the spot-leg of the trade. Also, a crucial difference with a collateralized loan such as a mortgage loan is that the seller (the cash lender) usually takes possession of the asset during the length of the repo.

The general view is that repos are used by market participants to obtain fundings, to gain access to specific securities to settle a trade, to hedge, or to support their market-making activities. However repos are hardly necessary to achieve these goals: By definition, repo market participants have sufficient resources to buy the assets they need, as otherwise they would not be able to conduct the spot leg of the repo. This begs the question: If they can buy the asset (be it cash or a security), why do they seemingly prefer to rent it?

The usual answer for a cash repo – the borrower uses repos to get cash – is that it is a collateralized loan. We find this answer unsatisfying: Contrary to most commercial loans, the securities pledged as collateral are highly liquid securities such as treasury securities. If counterparty risk was the issue, then the borrower would escape the haircut and get a better deal by just selling his collateral. So what frictions can explain that the borrowers prefer to rent his securities out for cash rather than just selling it? A plausible explanation is that cash borrowers have private information on the quality of their collateral which makes it less liquid. Although this would explain the presence of haircuts and why borrowers would prefer to pledge their assets as collateral, it does not explain why a large fraction of repo transactions are conducted with highly liquid securities that are unlikely to be subject to private information. So, why do market participants rent an asset when they can buy it?

In this paper, we present an environment with bilateral trades where the agents' utility of holding an asset is stochastic. We show that agents trading in a Walrasian market would

[1] Duffie (1996) describes the functioning of securities lending markets at length. In general it can be difficult to determine whether a trade is motivated by the funding needs of the security lender, by the security needs of the borrower, or both – although it seems clear that the Fed fund market which is based on repo originates from the needs of some market participants to obtain fed funds. Various estimates point to the very large size of the securities lending market. According to a survey conducted by ICMA (2012), securities lending was 17.1% of all repos conducted in 2011, or more than \$1.3 trillion (EUR1 trillion). Based on data from Data Explorer, the Financial Stability Board (FSB, 2012) provides another estimate: The total securities on loan globally would amount to about \$1.8 trillion as of April 2012, and if one also includes the inter-dealer repo market then this estimates is \$4 trillion. However, we should stress that these are only estimates as most repos are conducted over the counter.

find repos and asset sales redundant. So we introduce pairwise meeting and trading as a way to prevent the emergence of a Walrasian market outcome.[2] As a result of pairwise trading agents prefer to hold an (endogenous) level of securities in their portfolio. Although there are times when they do not necessarily need the securities, uncertainty implies that they prefer to hold on to them. Hence they rend it out when they do not need it, as this is a way to increase their profit without jeopardizing their long-run portfolio. They do not sell the asset in the market as they recognize the existence of a hold-up problem with bilateral trades: The next trader may be able to extract more when the agent wants to buy the asset back than what he is able to get for it today. As a result, agents are not indifferent between repo and sales and they optimally combine trades on both markets.

The explanation for this result is intuitive. In a Walrasian environment, the value of your asset is set by its price, and this price is the same independently of who you are, who you meet, of your asset holdings, or the asset holdings of others. Suppose you are craving for the asset today, but not tomorrow, and you are trading on a Walrasian market. Then you can buy a lot of the asset at the market price today, enjoy the benefits of holding it today and sell it back at the given market price tomorrow. This is impossible if you trade over the counter, because the current value of your asset holdings depends on your willingness to hold on to them (your current utility), but also on the willingness of your trading partner to pay for them, itself a function of his asset holdings and his utility. Suppose again you are craving for the asset today, but not tomorrow, and you are contemplating buying a lot of it over the counter. This may satisfy your craving today, but you will want to unload some of it tomorrow. The conditions under which you can unload your asset holdings will depend on who you meet and his asset holdings. Therefore, although you are hungy for the asset today, you may choose to buy a smaller amount of the asset than on a Walrasian market, for fear of getting a bad deal tomorrow. But then you do not consume the efficient level of the asset service. Renting the asset allows you to achieve efficiency without jeopardizing your future consumption.[3] We show that our results are robust to different fiscal treatments: If there is

[2] More generally, we assume that agents are unable to trade within a group with a aggregate valuation identical to the Walrasian market valuation.

[3] In other words, pairwise meeting creates a wedge between the present and the discounted future valuation (or price) of the asset, while there is none in a Walrasian market. Therefore, your current and future valuations will typically differ from the ones of your trading partner. But any efficient allocation will equate both the current and the future marginal valuations of asset holdings across agents, and this is only possible if there are two instruments: The outright purchase of assets and repo. Repos are efficient because they allow agents to attain a level of consumption which depends on their current valuation of the asset, independently of the uncertainty about their future valuation.

a tax on repos they are still useful as long as the tax or the subsidy is not too large.

We present a search environment but the results are not due to traders being matched at random. In fact, we consider both random and directed matching, in the sense of Corbae, Temzelides and Wright (2003). With directed matching, we impose a matching rule that, in equilibrium, maximizes the possible surplus from trade. Although agents have a good idea of whom they will meet in the future, they still find repo useful above and beyond the mere acquisition of assets. Also under this matching rule we show that, with two valuation shocks, an invariant distribution of assets has a two point support: There is one asset holding for each valuation. The difference in these two points is increasing in the persistence of valuation shocks: As they become more persistent, agents' asset holdings diverge. Inversely, as switching becomes more likely, agents tend to hold the same amount of the asset.

Still, there is some degree of randomness in the following sense: While an agent knows that he will meet a given type of traders, he ignores who he will meet within this type. Using a simple example, we show that repos become redundant if an agent can be matched with the "right" individual on *and* off the equilibrium path. Therefore, repos become essential to achieve the efficient allocation whenever a trader who decided to purchase more than he should have (relative to the equilibrium) is unable to reach the seller in the future (or the seller is unwilling to trade). This is the extent to which there are search frictions in our environment.

Also, our results are not due to the inability of agents to find a trading partner. Contrary to other papers in the related literature, agents always meet somebody they can trade with. Rather, the friction in this paper is the fact that matching and trading is bilateral. To convince the reader of this, we compute the equilibrium with increased matching speed, as matching speed may be seen as similar to a search friction. We show that agents repo less and less as the time to the next match decreases. However, as we drive the time to the next match to zero, we find that agents still use repos. The reason is that the outcome of the match is still depending on the asset holdings of the agents in the match, hence the wedge between present and future valuation is still there, which gives rise to the simultaneous usefulness of asset sale and repo.

In most of the cases of our model, agents are ex-ante identical and can alter between the "borrower" and "lender" sides of the repos market. But in the real world, some participants always appear on one side of the repos market. It is common to attribute this one-sided participation to legal/fiscal restrictions. However, consider the following scenario: A government encourages an insurance company to hold some government bonds. When presented with an offer to buy some bonds, why doesn't this insurance company sell and then purchase the bonds back again later? The government surely only requires that the holding of bonds is *on average* at some given level. Here we show that when the price of the asset is a function of the insurance company's asset holdings, then this company will prefer to rent the asset out than to sell it. More precisely, we show that if agents have ex-ante heterogeneous valuations for the asset and face preference shocks, then they would still choose to use repo for temporary adjustment of their portfolio. And whenever they do repo, they would be always "borrower" or "lender".

Using our simple environment, we can analyze the impact of the repo market on asset sales. Precisely, one might wonder if asset sales and repo transactions are two substitute activities, in which case we should expect the repo and sales volumes to co-move negatively for a given set of traders. We find that sales volume is a hump-shaped function of the degree of persistence of the valuation shock. As persistence increases, there are two margins. First, we find an extensive margin: As persistence increases, the support of the distribution of asset holdings also increases. Also, with directed matching, agents who just switched their valuation from high to low are matched with those agents who switched valuation from low to high. Therefore, as the difference in their asset holdings grows, also does the gains from trade, so that they trade a larger amount of the asset. However, there is also an extensive margin: Since shocks are more persistent, fewer agents switch types so that the total volume of sales can either increase or decrease. As the extensive and intensive margins go against each other, the sales volume is hump-shaped. However, we find that the volume of repos is always decreasing with persistence. As the future valuation becomes uncertain, i.e. shocks are not persistent, agents are unwilling to change their position through asset sales, but they are willing to engage in repos. Therefore, the total volume of repo is decreasing with persistence and it is higher than total sales volume when the uncertainty is high (or persistence is low).

Related literature

Our model applies to cash as well as securities lending repo market. We first describe the market for securities lending, as some readers may not be familiar with it. There are two ways traders can acquire a security for a short term use: They can either engage in securities lending or conduct "specials" repo. "Specials" are described in detail in Duffie (1996). Securities lending and "specials" have different legal and fiscal characteristics, so that a trader may prefer one over the other, but their economic function is the same: They allow a trader to acquire a specific security temporarily.[4] There are various reasons why a trader needs to borrow a particular security, but generally the securities lent are needed to support a trading strategy or a settlement obligation. These motivations are further analyzed in CACEIS (2010), Duffie (1996) or Vayanos and Weill (2008), but for our purpose, it suffices to say that the security provides a service to the borrower that he values above and beyond its mere cash flows.[5]

To induce the lender of a security to trade, he usually obtains a lower repo rate than the prevailing money market rates, and invests the funds in money markets for a profit. The rights of the holder of a security acquired through a repo or securities lending are very similar: In a repo transaction, the buyer owns the collateral asset, he can re-use them during the term of the repo by selling the asset outright, "repoing" them or pledging them to a third party.[6] In a securities-lending transaction, the borrower gains the ownership title to the securities lent while the lender gains full ownership of the title to the securities (or cash) pledged as collateral.[7] Finally, both the repo and the securities lending markets involve trades negotiated bilaterally out of electronic trading platforms and their clearing is executed without the help of a central counterparty.[8] Therefore, repo markets for the purpose of getting access to a securities and the securities lending markets are very similar, and they look a lot like a market where borrowers are just renting the asset for a short period of time. From now on, we will refer to the securities lending market or the cash repo market as simply the repo markets.

The importance of the securities lending market is highlighted in the empirical work of Fleming and Garbade (2007), who study the behavior of dealers at the Fed's securities lending facility. This facility was initiated by the Fed to solve settlement fails (when dealers fail to deliver promised securities). As described in the paper, the Fed auctions securities at noon, after the period of greatest liquidity in the over-the-counter securities lending market, to

[4]See CACEIS (2010) for a description of the different characteristics of repos and securities lending.

[5]Borrower may need to cover a failed transaction in the course of their trading activity, or a short position, or they may need to deliver securities they have not yet purchased against the exercize of a derivatives contract, or they want to raise specific collateral, perhaps for another securities lending transaction.

[6]See Monnet (2011) for the economics of rehypothecation.

[7]As explained in CACEIS (2010), the borrower can re-sell the securities borrowed, voting rights are transferred along with the title. Althought the borrower, as owner of the securities, is entitled to the possible economic benefits associated with ownership such as dividends and coupons, he is under the contractual oboigations to make equivalent payments in all distributions paid during the terms of the trade to the lender.

[8]See Koeppl and Monnet (2010), Carapella and Monnet (2011), Carapella and Mills (2011) or Acharya and Bisin (2010) for some economic analysis of the role of central counterparties.

give dealers access to an additional source of securities. Dealers can bid to borrow particular Treasury securities, while providing other Treasuries as collateral. Also, Fleming, Hrung and Keane (2010) study the effects of the Term Securities Lending Facility. This facility was instituted by the Fed in March 2008 to alleviate the financing strains of some securities dealers. By enabling dealers to swap less liquid assets for Treasury securities, the Fed allowed dealers to conduct their business at a time when some securities lost their liquidity. Finally, while our paper attempts at explaning why repos are useful, we do not tackle explanations of the tri-party repo market. For details on this market, we refer the reader to Copeland, Martin, and Walker (2010) or Ennis (2011).

There are few papers explaining the usefulness of repo markets. As Siritto (2012) shows, private information on the quality of assets can be a factor. The very fact that the seller is willing to repurchase the asset is a guarantee that the asset is of good quality. However, this is hard to apply to Treasury securities or in a dynamic setting where agents learn the quality of the asset. In an environment with no commitment, Mills and Reed (2008) have argued that repos are useful in order to cover counterparty risk. However it is not clear why agents do not sell the collateral to obtain fundings, specially as the lender generally applies haircuts to the collateral.[9] Finally, Duffie (1996) argues that selling an asset involves different costs than the one when conducting repos. In some sense, we would like to have a deeper understanding for the origin of these transaction costs, without necessarily resorting to different fiscal treatments or the inability of some agents to own some class of assets (such as market mutual funds), although we acknowledge the importance of such frictions. In this paper, we show that agents do better by using repos and outright purchase, rather than one of the two alone, even when there is full commitment, the quality of the asset is known, there is no risk exposure, and no differential fiscal treatments across types of trade.

Our paper also builds on several strands of the literature. First and foremost, it is related to the recent literature on over the counter market initiated by Duffie et. al. (2005) and generalized by Lagos and Rocheteau (2009). In this literature, traders face search frictions that they circumvent by contacting intermediaries (dealers). Dealers have access to a centralized interdealers market where they can trade their asset holdings at the market price. In our paper, we abscond from dealers and we only consider the problem of traders facing search frictions, but as in Lagos and Rocheteau, we allow for arbitrary asset holdings.[10] Another

[9]See Lacker (2001) and Kehoe and Levine (2006) for models of collateralized debt. See Ewerhart and Tapking (2011) for a model with repo contracts and endogenous choice of collateral.

[10]We suspect that if traders have to trade through dealers with access to an interdealer market, then traders will use lending to extract some of the dealer's surplus.

important difference is that we do not introduce matching frictions: Our agents will meet for sure, but they will meet in pairs. This allows us to characterize an equilibrium distribution of asset holdings, although holdings can be arbitrary. As in Lagos and Rocheteau (2009), we obtain results on the distribution of assets. They find that more severe search frictions are associated with less dispersion in the equilibrium asset distribution. We find that, as it becomes more likely that traders will have to readjust their portfolios (i.e. increased uncertainty about future valuation) the distribution of asset holdings also becomes less dispersed. Duffie, et. al. (2002) extends Duffie (1996) to study securities lending rates in a dynamic context, where traders have different opinions regarding the underlying value of the security. In their model, borrowing takes place as traders can take short position, i.e. traders who believe that the security's price will decline in the future will borrow and sell it today with the plan of purchasing it cheaper later to reimburse the loan. The option to short a security will naturally bring the price of this security higher, but more surprisingly, higher than the value of the most optimistic trader. This effect however, as well as lending rates are decreasing through time. In our paper, we do not study the dynamics of the lending rate, but we rather concentrate on the reasons why lending is optimal in the first place.

Our paper is also related to the literature on the liquidity of capital markets in general. Lagos and Rocheteau (2008) study an economy where agents can pay with money (an intrinsically useless object) or capital that can be used in production. To give a role for cash, they assume that agents are anonymous. But this prevents lending (or renting) from taking place, as agents do not know who they would lend capital to. Independently, Ferraris and Watanabe (2008) consider a very similar framework, but assume that agents can pledge capital as collateral instead of paying with it. Both sets of authors find a very related result: When there is a lack of liquidity or when agents are credit constraint, capital carries a liquidity premium and agents tend to accumulate too much capital relative to its first best level. This effect is absent from our framework because agents can always pay so that there is no liquidity problem.

Our paper also relates to the literature on leasing capital goods and more precisely to the recent paper by Gavazza (2011). There the author studies the leasing and secondary markets for aircrafts. He shows (and empirically confirms) that operators that face more volatile productivity shocks are more likely to lease aircrafts than those with less volatile shocks. Similarly, we find that the repo volume is a decreasing function in the degree of shock persistence. However, our two models differ substantially: Gavazza (2011) uses a model that is closely linked to the airline industry, where frictions are monitoring and transaction costs.

In contrast, our only friction is that agents meet in pair. Our model is also related to the literature on leasing by financially constrained firms.[11] However, we want to stress that in our context, agents are not financially constraint and in some cases they still prefer to rent than to buy.

The paper proceeds as follows. In Section 2 we describe the environment. In Section 3 we characterize Walrasian allocations as the benchmark. In section 4, we provide two examples to illustrate the importance of pairwise matching and uncertainty about future valuation for the coexistence of asset sale and repo. In section 5, we describe general allocations attainable under pairwise meeting and bargaining. In section 6, we solve for the equilibrium with directed matching and solve for the equilibrium distribution and volumes in general. Section 7 analyzes some variations of the model to illustrate the essential mechanisms behind the results. Section 8 concludes.

2. The model

The model borrows elements from Koeppl, Monnet, and Temzelides (2012) and Lagos and Rocheteau (2009). Time is discrete and the horizon is infinite. Each period has two subperiods: A trading stage, followed by a settlement stage.[12] There is a continuum of anonymous agents. In each period, there is a measure $1/2$ of two types of agents, type h and type ℓ. The type of an agent switches with probability $1-\pi \in [1/2, 1]$ at the start of the trading stage. We appeal to the law of large numbers to guarantee that there is the same measure of each type in each period.

There is a long-lived asset in fixed supply A. As in Lagos and Rocheteau (2009), we associate this asset to a Lucas-tree: One unit of the asset yields one unit of some fruit in the settlement stage. Agents of type $i \in \{h, \ell\}$ derive utility $u_i(a)$ from holding a units of the asset.[13] For simplicity, we impose the following condition,

Assumption 1. *$u_h'(a) \geq u_\ell'(a)$ for all a.*

[11]See for instance Eisfeldt and Rampini (2009).

[12]This two stages could be merged in one, but it helps the exposition to consider two separate stages.

[13] There are several interpretations for this formulation: Lagos and Rocheteau argue that this is the utility derived from the tree's fruit. Duffie, Garleanu and Pedersen (2009) explain that these are preferences from liquidity, hedging or other benefits that holding the assets may yield.

Therefore, for a given level of asset holding, the agent with the high type has a higher marginal utility than the agent with the low type. To be concise, we will refer to agents of type h as agents h and to agents of type ℓ as agents ℓ.

In the trading stage, agents can agree to trade the asset, in which case the seller transfers the assets *and* the fruits in the settlement stage. Or agents can only agree to trade the fruit of the asset: Then the seller only transfers the fruits that it yields, while he maintains ownership over the asset. We interpret this second trade as a repo trade, as the buyer surrenders the asset back to the seller once he enjoyed the benefits of holding it this period (that is consuming the fruits).

While the trading stage can be seen as a market, there is no market in the settlement stage. There, agents are endowed with a production technology for the numeraire good and it costs them one unit of utility to produce one unit of this good. Agents also consume the numeraire good and derive one unit of utility for each unit they consume. The numeraire good will be the settlement asset. Therefore the settlement of the terms of trade does not generate any net utility gains.[14]

3. Benchmark walrasian market

We first consider the case where the trading stage is a Walrasian market. We denote by p^r the price of a repo and by p^s the price of the asset. We consider only stationary equilibrium so that these prices are the same in each period. An agent $i = h, \ell$ with asset holdings a has a value $W_i(a)$ of holding the asset, where $W_i(a)$ is defined recursively as

$$\begin{aligned} W_i(a) \quad &= \quad \max_{a_i, q_i^r} u_i(c_i) - d + \beta E_{k|i} W_k(a_i') \\ \text{s.t.} \quad & p^r q_i^r + p^s q_i^s \leq d \\ & c_i = a + q_i^s + q_i^r \\ & a_i' = a + q_i^s \end{aligned}$$

where $E_{k|i}$ denotes the conditional expectation operator on future types k given the current type of the agent is i. The agent repos q_i^r and purchases an amount q_i^s of the asset during the period. Therefore, his current consumption of the security's service is c_i while his asset holding for the next period is a_i'. Naturally the quantity of repos q_i^r does not enter in the

[14]The settlement stage opens after the trading stage and on the same day. Opening the settlement stage in the next day does not affect the results.

continuation valuation but only in the momentary utility $u_i(.)$. The first order and envelope conditions yield

$$u_i'(c_i) + \beta E_{k|i} W_k'(a_i') = p^s \tag{1}$$

$$u_i'(c_i) = p^r \tag{2}$$

$$W_i'(a) = p^s \tag{3}$$

Notice from (3) that all agents value an additional unit of the asset in the same way when they enter the Walrasian market, independent of their type or of their asset holdings. There are two reasons for this: First, the utility is linear in the numeraire good such that there is no wealth effect in this model and, second, agents are playing against the whole market. In the next section, we will modify the latter. For the time being, the equilibrium prices and quantities satisfy,

$$\begin{aligned} (1-\beta)p^s &= p^r \\ u_h'(c_h) = u_\ell'(c_\ell) &= p^r \\ c_h + c_\ell &= 2A \end{aligned}$$

The first equation is a no-arbirtrage condition: Agents have to be indifferent between conducting a repo, in which case they have to pay the price p^r in terms of the numeraire good, and buying the asset at price p^s and reselling it in the next period at price βp^s. These two schemes are payoff equivalent and so should be their cost. As a consequence, anything goes for repos, and in particular $q_h^r = q_\ell^r = 0$. In other words, in a Walrasian market, there is no difference between conducting a repos or buying and selling the asset: Repos and outright purchases are perfect substitute. Therefore absent any additional frictions, the Walrasian benchmark cannot explain why repo market exists. In the following section, we depart from the Walrasian benchmark by assuming that agents trade bilaterally (or over the counter) in which case they bargain over the allocation.

4. An example

In this example, we illustrate the main mechanism at work in the general set-up by showing how *future* pairwise matching can explain a mix repos/sales today, even though 1) agents trade on a Walrasian market *today* and 2) agents know who they will meet tomorrow.[15] We consider a two period economy, where agents are endowed with asset holding A in the first period. Also in the first period, agents have access to a Walrasian market where they can both sell and/or repo the asset. Agent $i = h, \ell$ derives utility from consuming fruits in both periods and producing the numeraire good according to the utility

$$U_i(c^1, d^1, c^2, d^2) = u_i(c^1) - d^1 + \beta\pi \left[u_i(c^2) - d^2\right] + (1-\pi)\left[u_{i'}(c^2) - d^2\right],$$

where $u_h(c)$ and $u_\ell(c)$ satisfy Assumption 1. Since the world ends at $t = 2$, there is no difference between repo and sales then, as only the consumption of the fruits matters. However, we will show that pairwise matching in the future can explain the mix repo/sales *today*, in the Walrasian market.

Let us first consider the problem of a type i agent with asset holding A in the first period with access to the asset market at price p^s and repo market at price p^r. Then denoting the expected value for agent i of holding asset a_i at date 2 by $V_i(a_i)$, the problem of an agent i is

$$\begin{aligned} \max_{q^s, q^r} \quad & u_i\left(c_i^1\right) - d^1 + \beta V_i(a_i') \\ s.t. \quad & p^r q^r + p^s q^s \leq d^1 \\ & c_i^1 = A + q^s + q^r \\ & a_i' = A + q^s \end{aligned}$$

Using the first order conditions for the two types we obtain $c_i^1 = c_i^*$ and $a_i' = a_i^*$ for $i = h, \ell$ where

$$u_\ell'(c_\ell^*) = u_h'(c_h^*) \tag{4}$$

$$V_h'(a_h^*) = V_\ell'(a_\ell^*) \tag{5}$$

where the resource constraint is $c_\ell^* + c_h^* = a_h^* + a_\ell^* = 2A$. Prices satisfy $p^s = p^r + \beta V_i'(a_i^*)$ and $p^r = u_i'(c_i^*)$. As the utility functions are concave, there is a unique pair (c_h^*, c_ℓ^*) that satisfies (4) and the resource constraint. Similarly, if V_i is concave, there is a unique pair (a_h^*, a_ℓ^*) that satisfies (5) and the resource constraint. Notice that the repo market is active whenever $a_i^* \neq c_i^*$. This is what we will show below by solving for the functions $V_i(a)$.

[15] In the Appendix, we consider another example to illustrate how pairwise matching today (rather than tomorrow) can explain the usefulness of repos.

In period 2, agents trade bilaterally. The matching rule specifies that all agents h are matched with an agent ℓ and agents who switched types are matched together. Given the distribution of asset holdings has a two-point support at the end of period 1, this matching rule implies that a type i agent with asset holding a, will be matched with a type $j \neq i$ agent with asset holding $2A - a$. Also in equilibrium agents know exactly who they will meet. We assume that the agents h and ℓ bargain over the terms of trade so that the consumption levels c_h^2, c_ℓ^2 and the payment d are the solutions to the following bargaining problem at $t = 2$

$$\max \left[u_h(c_h^2) - d^2 - u_h(a_h)\right]^\theta \left[u_\ell(c_\ell^2) + d^2 - u_\ell(a_\ell)\right]^{1-\theta}$$

subject to $c_h^2 + c_\ell^2 = a_h + a_\ell = 2A$. Let us emphasize that a_h is not necessarily equal to a_h^*: Indeed, some agents ℓ holding a_ℓ^* have become h-agents, so that some h-agents are holding a_ℓ^* (and similarly for agents ℓ). Still, the matching is such that in all pairs $a_h + a_\ell = 2A$. The first order conditions give us

$$u_h'(c_h^*) = u_\ell'(c_\ell^*)$$

where $c_h^* + c_\ell^* = 2A$, and d^2 satisfies

$$d^2 = (1-\theta)\left[u_h(c_h^*) - u_h(a_h)\right] + \theta\left[u_\ell(c_\ell^*) - u_\ell(a_\ell)\right].$$

Therefore, the consumption of agents of type i in period 2 is efficient, as in period 1. Using these first order conditions, we obtain the usual expression for the payoff of an agent of type i holding a units of the asset, $v_i(a)$ for $i = h, \ell$,

$$v_h(a) = u_h(a) + \theta S(a, a_\ell) \quad (6)$$

$$v_\ell(a) = u_\ell(a) + (1-\theta) S(a_h, a) \quad (7)$$

so that agents split the surplus $S(a_h, a_\ell) = u_h(c_h^*) + u_\ell(c_\ell^*) - u_h(a_h) - u_\ell(a_\ell)$ according to their bargaining power. Notice that, contrary to the case with a Walrasian market, the payoff is not linear in asset holdings, but naturally depends on both agents' asset holdings. In particular, a quick inspection of (6) and (7) and the bargaining problem reveals that the marginal payoffs at $t = 2$ are pinned down by how the asset allocation affects the agents' outside options.

We can now write the expected value for agent i of holding asset a_i at date 2 as,

$$V_i(a_i) = \pi v_i(a_i) + (1-\pi) v_{-i}(a_i)$$

where we use the usual convention that $i \neq -i \in \{h, \ell\}$. Clearly, the marginal payoffs are sensitive to asset holdings since $V_i''(a) < 0$, which confirms our initial guess that (5) was giving a two-point support distribution for the asset holding at the end of the Walrasian market. However, using (6) and (7) it is easy to see that $V_i'(a)$ is a scaled weighted average of the marginal utility of each agent's type, or

$$V_i'(a) = u_i'(c_i^*) + \pi(1-\theta_i)\left[u_i'(a) - u_i'(c_i^*)\right] + (1-\pi)\theta_i\left[u_{-i}'(a) - u_i'(c_i^*)\right]. \tag{8}$$

with the natural convention $\theta_h = \theta$ and $\theta_\ell = 1-\theta$. If $\pi < 1$ then (1) and (8) evaluated at $a = c_i^*$ imply

$$V_\ell'(c_\ell^*) > u_\ell'(c_\ell^*) = u_h'(c_h^*) > V_h'(c_h^*) \tag{9}$$

so that at (c_h^*, c_ℓ^*) there is a wedge between the value of present and future consumption.[16] A quick inspection of (8) reveals why this is so: If an ℓ- agent holds c_ℓ^* he may switch to being a high type with probability $1-\pi$. In this case, he holds too little assets in which case his marginal utility is higher. A similar intuition holds for the h-agent.

Since the marginal continuation utility $V_i'(.)$ across the two types are not equated at the optimal consumption levels c_h^* and c_ℓ^*, the current marginal utility $u_i'(.)$ cannot be the same at the optimal asset holdings a_h^* and a_ℓ^*. Therefore a_i^* cannot equal c_i^* for $i = h, \ell$ and as a consequence both the repo market and the sales market are active at $t = 1$. If $\pi = 1$, there is no wedge in the sense that $V_i'(c_i^*) = u_i'(c_i^*)$ and in this case $a_i^* = c_i^*$: The repo market is inactive.

Figure 1 illustrates the choice of c_h^* and c_ℓ^* as well as why the repo market is active whenever the inequalities in (9) hold. On the left, Figure 1 shows the intratemporal indifference curves for $u_i(a) - d_i$ for the two types of agents, where in equilibrium $d_\ell = -d_h$. These indifference curves are tangent at the equilibrium consumption level (c_h^*, c_ℓ^*) where $c_h^* + c_\ell^* = 2A$ (the axis for d is reversed). On the right, Figure 1 super-imposes the intertemporal indifference curves $V_i(c) - d_i$. As (9) holds, the intratemporal indifference curve of the h- agent is steeper than its intertemporal indifference curve, and inversely for the ℓ-agent. Therefore, and as illustrated in Figure 1, the intertemporal indifference curves for both agents will be tangent at a point north-west of (c_h^*, c_ℓ^*). This explains why repos are useful: Asset sales alone are leaving unexploited gains from trade.

This example shows the importance for the argument of an agent's outside option in bargaining: It is this outside option that determines the marginal value of holding some

[16] The wedge equals $(1-\pi)\theta_i[u_j'(c_i^*) - u_i'(c_i^*)]$ and it is negative for h-agents and positive for ℓ-agents.

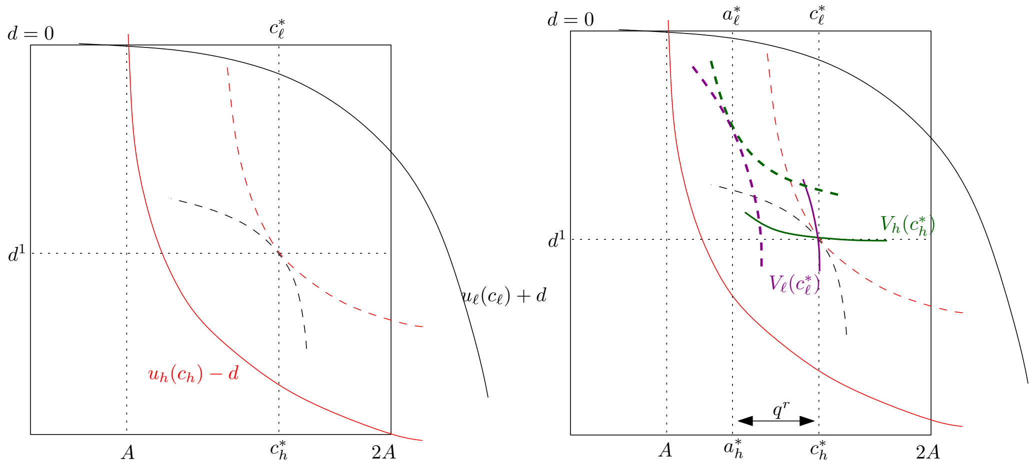


Figure 1: Active repo market

asset. Holding too little asset would give a bad outside option and, as $V_i''(a) < 0$, holding too much may cost too much relative to the additional benefits. This explains why agents do not want to take extreme positions, where they would repo or sell all their assets. Also, this example illustrates that it is pairwise trade that matters for the result: Indeed, agents are not randomly matched in this example. Below, we will show that this result holds true in the more general set-up.

5. Pairwise meeting and bargaining

We now assume that each agent h is matched with exactly one agent ℓ in the trading stage. We describe several matching technologies later. We consider allocations that are the solution to a bargaining game between both agents. We will consider a generic meeting between an agent h holding a generic amount of the asset a_h and an agent ℓ holding a generic amount of the asset a_ℓ. An allocation is a triple $\{q^s(a_h, a_\ell), q^r(a_h, a_\ell), d(a_h, a_\ell)\}$ where q^s denotes the quantity of the asset that the agent h buys from the agent ℓ (sells if negative), q^r is the quantity of the asset that the agent h buys or repo from the agent ℓ (sells or reverse repo if negative) and d is the numeraire transfer that the agent h makes in the settlement stage to the agent ℓ (receives if negative). We only focus on stationary and symmetric allocations. An allocation is feasible if

$$\begin{aligned} q^s(a_h, a_\ell) &\in [-a_h, a_\ell] \\ q^r(a_h, a_\ell) + q^s(a_h, a_\ell) &\in [-a_h, a_\ell] \end{aligned}$$

Notice that we do not allow short-selling. We will denote by $(\mathbf{q^s}, \mathbf{q^r}, \mathbf{d})$ the feasible allocations for all possible matches such that $(\mathbf{q^s}, \mathbf{q^r}, \mathbf{d})$ defines invariant distributions of asset holdings for agents h and ℓ. We denote these distributions by $\mu_i(a)$ for $i \in \{h, \ell\}$, where we have dropped the reference to the allocation for convenience. If they exist, a property of any invariant distribution is that

$$\frac{1}{2}\int a d\mu_h(a) + \frac{1}{2}\int a d\mu_\ell(a) = A$$

Then we can define recursively the expected value for agent $i \in \{h, \ell\}$ of holding asset a, before entering the trading stage, $V_i(a)$, as

$$\begin{aligned} V_h(a) &= \pi \int [u_h(c_h(a, a_\ell)) - d(a, a_\ell) + \beta V_h(a + q^s(a, a_\ell))] d\mu_\ell(a_\ell) \\ &+ (1-\pi) \int [u_\ell(c_\ell(a_h, a)) + d(a_h, a) + \beta V_\ell(a - q^s(a_h, a))] d\mu_h(a_h) \end{aligned} \tag{10}$$

where $c_h(a, a_\ell) = a + q^s(a, a_\ell) + q^r(a, a_\ell)$ is the consumption of the security's service of a type h with a units of the security matched with a type ℓ holding a_ℓ units of the security. Similarly, $c_\ell(a_h, a) = a - q^s(a_h, a) - q^r(a_h, a)$ is the consumption of the security's service of a type ℓ with a units of the security matched with a type h holding a_h units of the security. With probability π, an agent h remains an agent h. Then he meets an agent ℓ with asset a_ℓ according to the distribution μ_ℓ. Since he remains an agent h, he enjoys instant utility $u_h(.)$ from his asset holdings $a + q^s(a, a_\ell) + q^r(a, a_\ell)$ at the end of the settlement stage. However, he only carries $a + q^s(a, a_\ell)$ over to the next period since repos do not involve the transfer of the asset but only of fruits. The agent values this portfolio according to $\beta V_h(a + q^s(a, a_\ell))$. With probability $1 - \pi$ the agent h becomes an agent ℓ. In this case, he meets an agent h according to the distribution μ_h and he enjoys instant utility $u_\ell(.)$ from his asset holdings $a - q^s(a_h, a) - q^r(a_h, a)$. He values his remaining portfolio according to $\beta V_\ell(a - q^s(a_h, a))$. Similarly for agents ℓ,

$$
\begin{aligned}
V_\ell(a) &= \pi \int [u_\ell(c_\ell(a_h, a)) + d(a_h, a) + \beta V_\ell(a - q^s(a_h, a))] d\mu_h(a_h) \\
&+ (1-\pi) \int [u_h(c_h(a, a_\ell) - d(a, a_\ell) + \beta V_h(a + q^s(a, a_\ell))] d\mu_\ell(a_\ell)
\end{aligned} \tag{11}
$$

We assume that agents cannot commit to participate ex-ante and an allocation $(\mathbf{q^s}, \mathbf{q^r}, \mathbf{d})$ is individually rational if all agents prefer the allocation to being in autarky this period. That is, for any porfolio a, an agent h matched with an agent ℓ with a portfolio a_ℓ prefers the allocation than not trading today, i.e.

$$
u_h(c_h(a, a_\ell)) - d(a, a_\ell) + \beta V_h(a + q^s(a, a_\ell)) \geq u_h(a) + \beta V_h(a),
$$

and similarly for an agent ℓ matched with an agent h with portfolio a_h,

$$
u_\ell(c_\ell(a_h, a)) + d(a_h, a) + \beta V_\ell(a - q^s(a_h, a)) \geq u_\ell(a) + \beta V_\ell(a).
$$

From now on, for concision and whenever there is no risk of confusion, we will drop references to the agents' portfolios in an allocation.

With general Nash bargaining where the agent h has bargaining power $\theta \in [0,1]$, the allocation of an agent h with portfolio a_h matched with an agent ℓ with portfolio a_ℓ solves the following problem:

$$
\begin{aligned}
\max_{q^s, q^r, d} & [u_h(c_h) - d + \beta V_h(a_h + q^s) - u_h(a_h) - \beta V_h(a_h)]^\theta \\
& \times [u_\ell(c_\ell) + d + \beta V_\ell(a_\ell - q^s) - u_\ell(a_\ell) - \beta V_\ell(a_\ell)]^{1-\theta}
\end{aligned}
$$

subject to the allocation being feasible. The first order conditions for an interior solution[17] are

$$
V_h'(a_h + q^s) = V_\ell'(a_\ell - q^s) \tag{12}
$$

$$
u_h'(c_h) = u_\ell'(c_\ell) \tag{13}
$$

$$
\begin{aligned}
d(a_h, a_\ell) &= (1-\theta)[u_h(c_h) - u_h(a_h) + \beta V_h(a_h + q^s) - \beta V_h(a_h)] \\
&\quad - \theta[u_\ell(c_\ell) - u_\ell(a_\ell) + \beta V_\ell(a_\ell - q^s) - \beta V_\ell(a_\ell)]
\end{aligned} \tag{14}
$$

[17] In the case where $q^s = a_\ell$, (1) becomes $V_h'(a_h + q^s) > V_\ell'(a_\ell - q^s)$, while in the case where $q^s + q^r = a_\ell$, (13) becomes $u_h'(a_h + a_\ell) \geq u_\ell'(0)$.

Equations (1) and (13) characterize the allocations $q^s(a_h, a_\ell)$ and $q^r(a_h, a_\ell) = c_\ell - [a_\ell + q^s(a_h, a_\ell)]$. Inspecting (1) and (13) agents use repos whenever $c_h \neq a_h + q^s$ and $c_\ell \neq a_\ell - q^s$. Also notice that (13) together with $c_h + c_\ell = a_h + a_\ell$ uniquely defines c_h and c_ℓ. The transfer $d(a_h, a_\ell)$ redistributes the surplus from the trade according to the bargaining weights. Finally, notice that the allocation depends on the distributions of asset holdings μ_i for $i = h, \ell$ as they affect the value functions V_i. Therefore, to fully characterize the equilibrium with an invariant distribution, we need to specify how agents are matched. In the next section, we assume that agents direct their search and we study random matching later.

6. Directed search

We now describe a rather sophisticated matching technology. Following Corbae, Temzelides and Wright (2003), we use directed search: The matching function specifies that agents who switched types are matched together. Therefore a "new" agent h will be matched with a "new" agent ℓ.[18] Below we verify that this matching function is an equilibrium matching rule (where such a term is precisely defined). We devote the rest of this section to the following result.

Proposition 2. *With directed search, there is an equilibrium characterized by a degenerate distribution of asset holdings for each type at some level $\bar{a}_i$ with $i = h, \ell$ with $q^s(\bar{a}_h, \bar{a}_\ell) = 0$, $q^s(\bar{a}_\ell, \bar{a}_h) = \bar{a}_h - \bar{a}_\ell$ and $q^r(\bar{a}_h, \bar{a}_\ell) = q^r(\bar{a}_\ell, \bar{a}_h) = q^r$ where q^r solves $u_h'(\bar{a}_h + q^r) \geq u_\ell'(\bar{a}_\ell - q^r)$ (with equality if $q^r < \bar{a}_\ell$).*

In words, each type of agents is holding a specific portfolio, either $\bar{a}_h$ or $\bar{a}_\ell$ for type h and ℓ respectively. Agents who did not switch type just engage in repo. Agents who switched type adjust their asset holdings so that they hold their type's portfolio. Then they conduct repo as if they never switched. Loosely speaking, there is a sense in which agents first access the asset market and then engage in repo.

Suppose first that there is a pair $(\bar{a}_h, \bar{a}_\ell)$ such that the functions q^r and q^s specified in Proposition 2 satisfy conditions (12) and (13). To verify that they form an equilibrium we first need to verify that an agent would not prefer to be matched with a different agent than the one he is assigned to, or that no agent would prefer to interact with him to trading with his assigned agent. In the terminology of Corbae, Temzelides and Wright (2003), the proposed matching rule is an equilibrium matching if no coalition consisting of 1 or 2 agents

[18] It turns out that in equilibrium this matching function will maximize the gains from trade in each match, so that if agents could choose, they would actually want to be matched in this way, as we show below.

can do better (in the sense that the discounted lifetime utility of all agents in the coalition increases) by deviating in the following sense: An individual can deviate by matching with himself (i.e. being in autarky this period) rather than as prescribed by the matching rule; and a pair can deviate by matching with each other rather than as prescribed by the matching rule. It should be clear that the bargaining solution is always better than autarky (although not in a strict sense). Hence we only have to consider a deviation by two agents. However, notice that the matching rule maximizes the gains from trade for those agents who just switched. So these agents would be worse off if they were matched differently. It follows that the matching rule is an equilibrium matching rule.

We now show that given the functions q^r and q^s specified in Proposition 2, there exists an equilibrium support $\{\bar{a}_h, \bar{a}_\ell\}$ for the distribution of assets. We first consider the terms of trade for a repo and the sale of the asset. The "price" of the repo trade is $d(\bar{a}_h, \bar{a}_\ell)$, the transfer between an agent h holding $\bar{a}_h$ and an agent ℓ holding $\bar{a}_\ell$. Indeed, there is no sale of the asset in this match as agents h and ℓ keep the same asset holding. Now, the "price" of the repo combined with the asset sale transaction is $d(\bar{a}_\ell, \bar{a}_h)$, as the agent h and ℓ now change their asset position, but also trade fruits. In the Appendix, we show that

$$\begin{aligned} d(\bar{a}_h, \bar{a}_\ell) &= (1-\theta)[u_h(\bar{c}_h) - u_h(\bar{a}_h)] + \theta[u_\ell(\bar{a}_\ell) - u_\ell(\bar{c}_\ell)] \\ d(\bar{a}_\ell, \bar{a}_h) &= d(\bar{a}_h, \bar{a}_\ell) + \bar{u} + \beta(1-\theta)[V_h(\bar{a}_h) - V_h(\bar{a}_\ell)] + \beta\theta[V_\ell(\bar{a}_h) - V_\ell(\bar{a}_\ell)] \end{aligned}$$

where

$$\bar{u} = (1-\theta)[u_h(\bar{a}_h) - u_h(\bar{a}_\ell)] + \theta[u_\ell(\bar{a}_h) - u_\ell(\bar{a}_\ell)].$$

As expected, $d(\bar{a}_\ell, \bar{a}_h) > d(\bar{a}_h, \bar{a}_\ell)$, and the price of the repo combined with the asset sale is composed of the repo price, plus the weighted present and discounted lifetime gains of switching position for agents h and ℓ.

The directed matching technology specifies that an agent h with $\bar{a}_\ell$ meets an agent ℓ with $\bar{a}_h$ and an agent h with $\bar{a}_h$ meets an agent ℓ with $\bar{a}_\ell$. Given $q^s(\bar{a}_h, \bar{a}_\ell) = 0$, $q^s(\bar{a}_\ell, \bar{a}_h) = \bar{a}_h - \bar{a}_\ell$ and $q^r(\bar{a}_h, \bar{a}_\ell) = q^r(\bar{a}_\ell, \bar{a}_h) = q^r$, we obtain the following value functions,

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{c}_h) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{c}_\ell) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)] \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{c}_\ell) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{c}_h) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)] \end{aligned}$$

Notice that we need to specify the value of the outside option in order to solve for the bargaining solution in equilibrium, i.e. $V_h(\bar{a}_\ell)$ and $V_\ell(\bar{a}_h)$. To simplify the analysis we assume that the matching technology then specifies that an agent h holding $\bar{a}_\ell$ (respectively $\bar{a}_h$) who did not trade in the previous period is matched with an agent ℓ holding $\bar{a}_h$ (respectively $\bar{a}_\ell$), and reversely.[19] Therefore we obtain,

$$\begin{aligned} V_h(\bar{a}_\ell) &= \pi[u_h(\bar{c}_h) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{c}_\ell) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] \\ V_\ell(\bar{a}_h) &= \pi[u_\ell(\bar{c}_\ell) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{c}_h) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] \end{aligned}$$

Using these value functions we find that

$$d(\bar{a}_\ell, \bar{a}_h) = d(\bar{a}_h, \bar{a}_\ell) + \frac{\bar{u}}{1-\beta}$$

so that the value of selling the asset is just $\bar{u}/(1-\beta)$, the lifetime discounted surplus from adjusting portfolios. Notice that as $\pi \to 1/2$ we have $\bar{a}_h \to \bar{a}_\ell$ so that $\bar{u} \to 0$ and there is no value of selling the asset.

In the Appendix, we characterize the bargaining solution with directed matching q^r, $\bar{a}_h$ and $\bar{a}_\ell$.

Proposition 3. *The degenerate supports $\bar{a}_h$ and $\bar{a}_\ell$ of the two distributions with bargaining are fully characterized by the following equations,*

$$\begin{aligned} u_h'(\bar{c}_h) &= u_\ell'(\bar{c}_\ell) \\ u_h'(\bar{c}_h) &= \frac{[\pi - (2\pi-1)\beta][\theta u_\ell'(\bar{a}_\ell) - (1-\theta)u_h'(\bar{a}_h)] - (1-\pi)[\theta u_\ell'(\bar{a}_h) - (1-\theta)u_h'(\bar{a}_\ell)]}{(2\pi-1)(1-\beta)(2\theta-1)} \\ \bar{c}_h + \bar{c}_\ell &= \bar{a}_h + \bar{a}_\ell = 2A \end{aligned}$$

Inspecting these equations, notice that in the case with no persistence, $\pi = 1/2$ we obtain that agents h and ℓ hold the same amount of asset, $\bar{a}_\ell = \bar{a}_h = A$. In this case there is only repos and no asset sales, $q^s = 0$ and $q^r > 0$. This result is intuitive: When an agent's type today is unrelated to his type tomorrow, all agents have the same future value of holding the asset, irrespective of their current types, and so they equate their asset holdings. The reverse holds in the case with full persistence, $\pi = 1$. Then there is neither any repos nor asset sales: $q^r = q^s = 0$ and the distribution of asset satisfies $u_\ell'(\bar{a}_\ell) = u_h'(\bar{a}_h)$. Again, this is intuitive: When types are permanent, agents know that they always value the asset the same way. Therefore, they just hold the amount they need and they never trade. We can

[19]Notice that in equilibrium, there is always trade unless $\pi = 1$ in which case agents never switch type so that this ad-hoc rule is never used.

then vary the degree of persistence to obtain market volumes as a function of persistence.

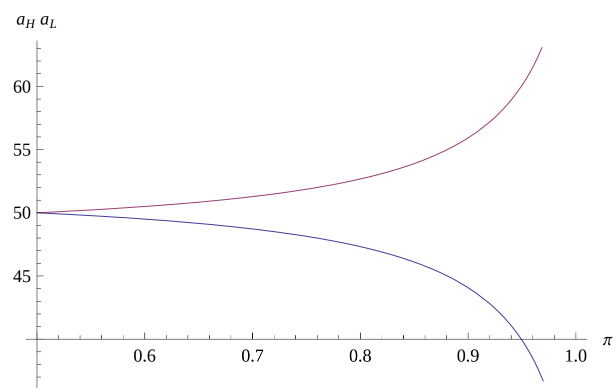


Figure 2: Asset holdings as a function of π.

Corollary 4. *$\bar{a}_h - \bar{a}_\ell \geq 0$ is increasing in π. Sales volume is hump-shaped in π while repos volume is strictly decreasing in π.*

With $\pi = 1/2$ we know there is no outright purchase, but only some repo to allocate the fruits to those agents h who like it the most. As π increases, it is more likely that an agent h becomes once again an agent h next period. Therefore his valuation for the asset increases, and starting from $a_h = a_\ell = A$, there are gains from trade when an agent h meets an agent ℓ. This is the extensive margin: As π increases, the difference in the equilibrium asset holdings of agents h and ℓ increases. Therefore, agents who just switched their valuation trade more. However, there is also an intensive margin: As π increases, fewer agents switch types and so fewer pairs of agents trade. The combination of the extensive and intensive margins explains why asset sales are hump-shaped in π. In contrast, the volume of repos is always decreasing with persistence. As π increases agents adjust their position to the optimal quantity of the asset they would like to hold absent any future uncertainty. Therefore they need less repo to achieve the efficient level of consumption as π increases. Therefore, the total volume of repo is decreasing with π and it is higher than total sales volume when $\pi = 1/2$. Finally, when $\pi = 1$, agents know their type for sure. Hence in equilibrium, all gains from trades (be it asset trade or fruit trade) are extinguished and there is neither sales nor repos.

Also, we can find the price for repo and asset sales.

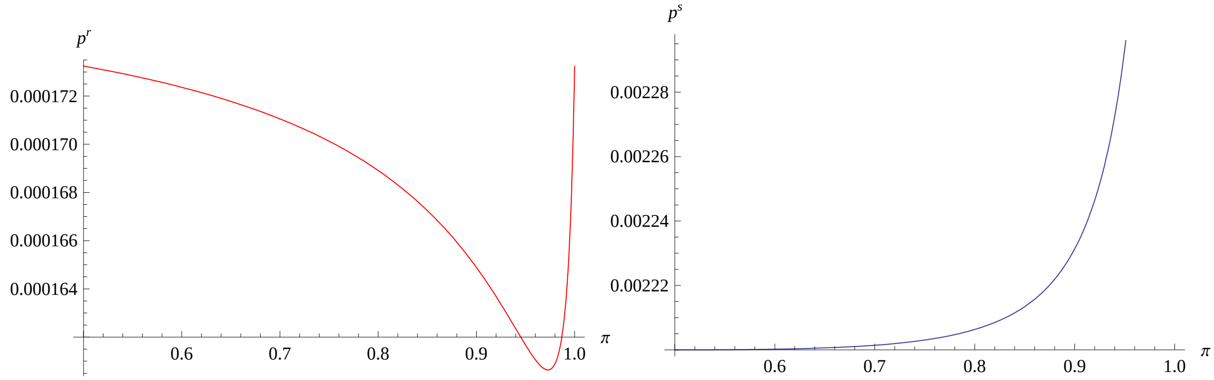


Figure 3: Repo prices (left) and outright purchase price (right)

Corollary 5. *Let p^r be the price of a repo and p^s the price of a sale. Then*

$$p^r = \frac{(1-\theta)[u_h(\bar{c}_h) - u_h(\bar{c}_h - q^r)] + \theta[u_\ell(\bar{c}_\ell + q^r) - u_\ell(\bar{c}_\ell)]}{q^r}$$

$$p^s = \frac{(1-\theta)[u_h(\bar{c}_h - q^r) - u_h(\bar{c}_\ell + q^r)] + \theta[u_\ell(\bar{c}_h - q^r) - u_\ell(\bar{c}_\ell + q^r)]}{(1-\beta)(\bar{a}_h - \bar{a}_\ell)}$$

We find these prices by using the definition of the payments $d(a_h, a_\ell)$. Since agents who did not swith types only conduct repos, we infer that $d(\bar{a}_h, \bar{a}_\ell) = p^r q^r$. Therefore,

$$p^r q^r = (1-\theta)[u_h(\bar{c}_h) - u_h(\bar{a}_h)] + \theta[u_\ell(\bar{a}_\ell) - u_\ell(\bar{c}_\ell)].$$

Also, since the pair of agents that switched conducts both an asset sale q^s to adjust their position, and then a repo. Therefore $d(\bar{a}_\ell, \bar{a}_h) = p^r q^r + p^s q^s$. And as $d(\bar{a}_\ell, \bar{a}_h) = d(\bar{a}_h, \bar{a}_\ell) + \bar{u}/(1-\beta)$ we obtain that

$$p^s q^s = \frac{\bar{u}}{1-\beta}.$$

Using the expression for $\bar{u}$, with $q^s = \bar{a}_h - \bar{a}_\ell$, we get the result.

Figure 2 shows how $\bar{a}_h$ (red curve) and $\bar{a}_\ell$ (blue curve) evolve as π varies from 1/2 to 1. The parameters chosen are $\theta = 0.5, \lambda = 0.1, \sigma = 2, \beta = 0.9,$ and $A = 50$. Interestingly, the rate of divergence increases as types become more persistent. Hence, as π becomes large, we should expect some movements in prices and quantities. This intuition is confirmed by Figure 3 that shows prices for repo p^r and asset sales p^s.

Similarly total repo volume q^r and total sales volume $(1-\pi)q^s$ display very different pattern, as illustrated in Figure 4. At $\pi = 0.9$, the total volume of repo is approximately 20% of the outstanding securities, while total sales are only 1% of outstanding securities.

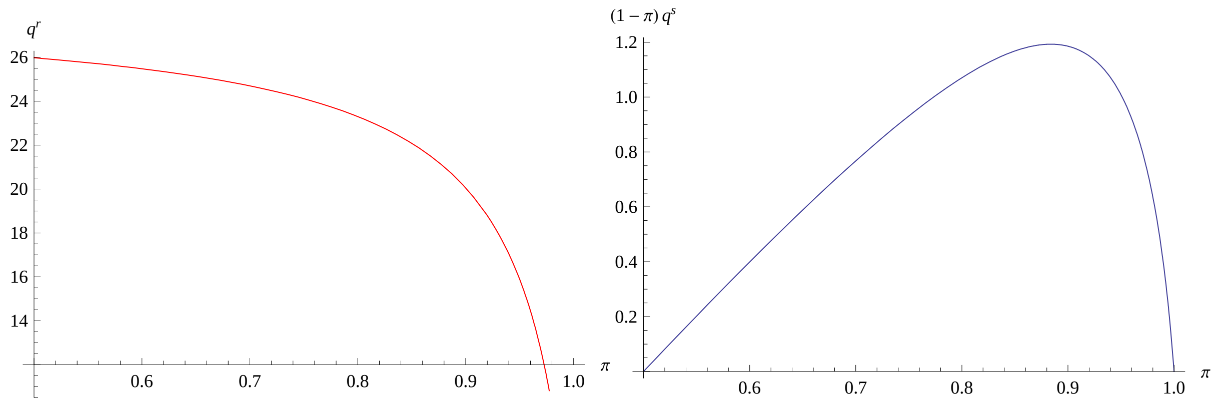


Figure 4: Repo volumes (left) and outright purchase volumes (right)

The coefficient of risk aversion σ is the one with the most impact on asset volumes and values. For the sake of illustration and without any intention of being a serious numerical exercize, with $A = 50$, we can match the observations on repos and sales of Treasury securities, with $\sigma = 0.5$ (close to risk neutrality) and a fairly high persistence, $\pi = 0.9$.[20]

7. Some variations on the model

In this section we consider several extensions to the basic framework. First, we consider the effect of agents becoming more patient on the existence of repos. Then we consider whether their ability to trade often in between their utility shocks affects the existence of repos. We also consider a different option for future trade: At the moment, agents always had to trade bilaterally, but in this section we will study the existence of repo when agents can choose to trade on a Walrasian market rather than over the counter. Then we study the equilibrium with random matching instead of directed matching. After that, we study the effect of a wedge between the cost of trading a repo and the one for an outright purchase. Finally, we study the case with ex-ante asymmetric agents to show how some participants might only appear as "borrower" or "lender" in the repos market.

7.1. Patience

It is clear that in general those prices in Corollary 5 are different from their Walrasian equivalent, and in particular that $(1-\beta)p^s$ is different from p^r. However, an interesting

[20]The average daily volume of Treasury repos is approximately twice the one for Treasury sales in the US according to ICAP, see http://www.icap.com/investor-relations/monthly-volume-data.aspx.

case to consider is when agents become very patient. Then it is legitimate to guess that the allocation will converge to the Walrasian one, as it is in some sense equivalent to agents trading with each other very frequently. However, it is also as if agents were also changing type very often and although we have the illusion that they can trade very fast whenβ converges to one, they are also bargaining a lot to readjust their protfolio and this friction remains. Indeed, as β tends to one, the solution to the bargaining problem is characterized by $\bar{a}_h, \bar{a}_\ell \to A$, so that asset sales converge to zero. Hence, we obtain

$$\lim_{\beta\to 1}(1-\beta)p^s = (1-\theta)u_h'(A) + \theta u_\ell'(A).$$

However in the limit q^r satisfies $u_h'(A+q^r) = u_\ell'(A-q^r)$ and Assumption 1 guarantees that $q^r > 0$ is bounded away from zero. Since $q^r > 0$ and $u_i(.)$ is concave,

$$q^r u_h'(A+q^r) < u_h(A+q^r) - u_h(A) < q^r u_h'(A)$$
$$q^r u_\ell'(A) < u_\ell(A) - u_\ell(A-q^r) < q^r u_\ell'(A-q^r)$$

and in general $p^r \neq (1-\beta)p^s$. For illustration, we use the following utility function: $u_h(a) = \frac{a^{1-\sigma}}{1-\sigma}$ and $u_\ell(a) = \lambda u_h(a)$ where $\lambda \in (0,1)$. Then we obtain

$$q^r = \frac{\lambda^{-\frac{1}{\sigma}}\bar{a}_\ell - \bar{a}_h}{1+\lambda^{-\frac{1}{\sigma}}}$$

so that

$$\bar{a}_h + q^r = \lambda^{-\frac{1}{\sigma}}\frac{2A}{1+\lambda^{-\frac{1}{\sigma}}} \text{ and } \bar{a}_\ell - q^r = \frac{2A}{1+\lambda^{-\frac{1}{\sigma}}}$$

Figure 5 shows how repo volume moves along the (β,π)-dimension.

As we have argued above, a_ℓ and a_h tends to A whenever $\beta \to 1$. Therefore in this case,

$$\lim_{\beta\to 1}(1-\beta)p^s = (1-\theta+\lambda\theta)A^{-\sigma},$$

while

$$\lim_{\beta\to 1} p^r = \frac{A^{-\sigma}}{1-\sigma}\frac{\left(\lambda^{\frac{1}{\sigma}}+1\right)^{\sigma}}{\left(1-\lambda^{\frac{1}{\sigma}}\right)}[2^{1-\sigma} - \left(\lambda^{\frac{1}{\sigma}}+1\right)^{1-\sigma}]\left\{1-\theta+\lambda\theta\frac{\left(\lambda^{\frac{1}{\sigma}}+1\right)^{1-\sigma} - \left(2\lambda^{\frac{1}{\sigma}}\right)^{1-\sigma}}{\lambda 2^{1-\sigma} - \lambda\left(\lambda^{\frac{1}{\sigma}}+1\right)^{1-\sigma}}\right\}.$$

With $\lambda = 0.1$ and $\sigma = 2$, we plot the ratio $\lim_{\beta\to 1}(1-\beta)p^s/\lim_{\beta\to 1}p^r$ as a function of

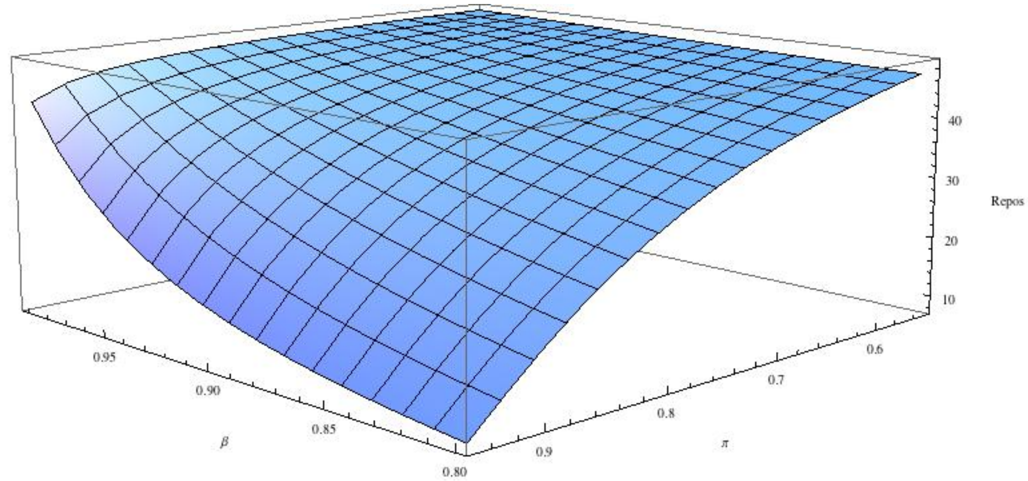


Figure 5: Repo volume in the (β, π)-space.

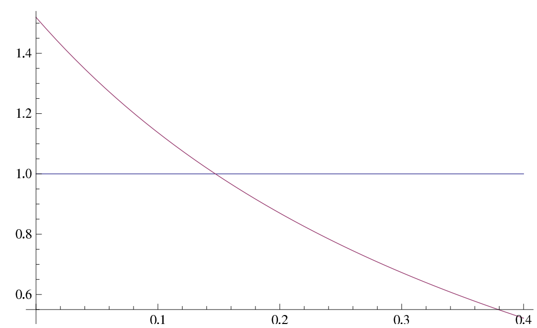


Figure 6: Ratio of repo and purchase prices

$\theta \in [0, 0.4]$. As Figure 6 shows, $\lim_{\beta \to 1}(1-\beta)p^s > \lim_{\beta \to 1} p^r$ for low values of θ and the inequality is reversed otherwise. In the next section we correct for the frequency with which agents change type as they can trade more often.

7.2. Frequent trades

In this subsection we study the consequences of agents meeting more frequently. More specifically, what are the consequences of reducing the time until the next meeting from one unit to $\Delta < 1$ units? And, what happens when $\Delta \to 0$? If β denotes discounting over a period of unit length, and π denotes the probability of maintaining the same type over a period of unit length, we assume agents discount future at rate $\beta_\Delta = 1 - \Delta(1-\beta)$ and the probability of maintaining the same type is $\pi_\Delta = 1 - \Delta(1-\pi)$ over a period of $\Delta < 1$ unit length. Clearly the level of consumption by the type h and ℓ agents will remain the same, however, the share of asset reallocation via repo and sale changes. We denote the repo level when $\Delta < 1$ units of time elapses until the next meeting by q^r_Δ, and we show in the Appendix,

Proposition 6. *For any $\Delta < 1$, we have $q^r_\Delta < q^r$. And as $\Delta \to 0$, q^r_Δ decreases to $q^r_0 > 0$.*

That is even with very frequent trades agents use repo. Although agents can trade more often they still face the friction that trading has to be bilateral and this gives a role for repo.

7.3. Outside option

An intuitive explanation for our results, reminiscent of the intuition from our earlier example, is that agents may prefer to use repos (to acquiring an asset), because they do not want to lock in a position that may be difficult to undo later at an agreeable price. When they engage in repos, agents are not locked into a position. To make this intuition more precise, we modify the environment slightly and assume that agents' outside option is to access a Walrasian market from next period onward. Then the outside option for an agent i holding a units of the asset is $u_i(a) + \beta\tilde{W}_i(a)$ where $\tilde{W}_i(a) = \pi W_i(a) + (1-\pi)W_{-i}(a)$ and $W(a)$ has been defined in Section 3. The possibility to trade on a Walrasian market would make the "lock-in" problem a little less severe, as agents could sell their securities on the walrasian market next period. Therefore we would expect the repo trade to decrease relative to the economy where agents do not have the option to unload their asset holdings on a Walrasian market. Still, agents are locked-in for one period and we would still expect repo to have a role. Indeed, let $\bar{q}^r$ be the equilibrium level of repo taking place in the economy where agents have the option to trade at Walrasian price in the next period, and let q^r be the equilibrium level of repo when they do not have this option. Then, in the Appendix, we show

Proposition 7. *With directed search and bargaining, there is an equilibrium where $q^r > \bar{q}^r > 0$.*

The equilibrium with the option to trade on a Walrasian market displays the same features as the one in our original set-up. That is, whether agents switched types or not, they always repo $\bar{q}^r$. In addition, those agents who switched types trade $\bar{q}^s = \bar{a}_h - \bar{a}_\ell$ and zero otherwise, where $\bar{a}_h$ and $\bar{a}_\ell$ are given by some equilibrium conditions. The important result is that, although agent's outside option is the Walrasian price (next period), agents will still use repos, but less so than if they did not have the option to trade at Walrasian price the following period. Therefore, the repo volume declines as we take the economy "closer" to its Walrasian benchmark.

7.4. Random matching: special cases

Here, we study two extreme cases with either $\pi = 1/2$ or $\pi = 1$ and an agent h is randomly matched with an agent ℓ. In the case with $\pi = 1/2$, preference shocks have no persistence and current preferences do not give any information on future preferences. In the case with $\pi = 1$ preference shocks are fully persistent as they are fixed forever.

With no persistence and random matching, we obtain the following result.

Proposition 8. *With random matching and $\pi = 1/2$, there is a unique invariant equilibrium characterized by a distribution of asset holdings for each type that are degenerate at some level $\bar{a} = A$ with $q^s(\bar{a}, \bar{a}) = 0$, and $q^r(\bar{a}, \bar{a}) > 0$.*

In the case without persistence, (10) and (11) imply that $V_h(a) = V_\ell(a)$ for all a, such that agents h and ℓ enjoy the same value of holding the asset. In this case, (1) implies that $a_h + q^s(a_h, a_\ell) = a_\ell - q^s(a_h, a_\ell)$ with $q^s(a_h, a_\ell) > 0$ if and only if $a_\ell > a_h$ and $q^s(a_h, a_\ell) < 0$ otherwise. That is, agents leave the match holding the same quantity of asset. Hence, the unique invariant equilibrium is one where the distribution of asset holding is degenerate at $\bar{a} = A$ and $q^s(\bar{a}, \bar{a}) = 0$. This is very intuitive: Since all agents give the same value to future returns, they extinguish all surplus from trading the asset by averaging their asset holding (i.e. once an agent holding a_h trade with an agent hoding a_ℓ, they both end up with $(a_h + a_\ell)/2$) and in equilibrium they hold the same amount of the asset. Then (13) together with Assumption 1 imply that $q^r(\bar{a}, \bar{a}) > 0$: While agents value future asset returns the same way, they differ in their valuation of current return. Therefore, there is a benefit from repos, where only the current return is traded.

With full persistence however, there is an equilibrium with neither asset sales nor repo in equilibrium.

Proposition 9. *With random matching and $\pi = 1$, there is an equilibrium with a degenerate distribution of asset holdings for each type at some level $\bar{a}_h$ and $\bar{a}_\ell$ with $\bar{a}_h > \bar{a}_\ell$ where $q^s(\bar{a}_h, \bar{a}_\ell) = 0$ and $q^r(\bar{a}_h, \bar{a}_\ell) = 0$.*

We will first verify that the proposed allocation is an equilibrium. Since $q^s(\bar{a}_h, \bar{a}_\ell) = 0$ and $q^r(\bar{a}_h, \bar{a}_\ell) = 0$, equation (14) implies that $d(\bar{a}_h, \bar{a}_\ell) = 0$. Using (10) and (11), we then have for $i = h, \ell$,

$$V_i(\bar{a}_i) = \frac{u_i(\bar{a}_i)}{1-\beta} \tag{15}$$

and (1) and (13) imply that $\bar{a}_h$ and $\bar{a}_\ell$ are uniquely given by

$$u'_h(\bar{a}_h) = u'_\ell(\bar{a}_\ell)$$

with $\bar{a}_h = 2A - \bar{a}_\ell$. This verifies that there is no asset sales or repos in equilibrium. Also, combining the last equation with Assumption 1, we can verify that $\bar{a}_h > \bar{a}_\ell$. This equilibrium is unique whenever endowments are symmetric across all agents (and no constraint binds – which may happen if some agents ℓ are endowed with too many securities in the first place) so that all agents ℓ hold the same amount a_ℓ and all agents h holds a_h. To see this notice that if an agent h endowed with a_h meets an agent ℓ endowed with a_ℓ, then the bargaining solution imposes that they trade so that (13) holds. But the unique solution is $a_h + q^s(a_h, a_\ell) = \bar{a}_h$ and $a_\ell - q^s(a_h, a_\ell) = \bar{a}_\ell$. Since $a_h + a_\ell = 2A$, such a q^s exists and takes the agents directly to the equilibrium distribution of asset holdings.

For general levels of persistence $\pi \in (0, 1)$ and random matching, we are unable to determine analytically the total volume of sales and repos as we cannot solve analytically for the invariant equilibrium distribution of asset holdings.[21] We suspect that as an agent ℓ is better endowed, he will sell more to agent h, and as agent h is less endowed, he will buy more from agent ℓ. This would hint to more trade as agents valuations differ and we would expect that the distributions of asset holdings become more spiked around their respective mean $\bar{a}_h$ and $\bar{a}_\ell$ as π increases, where the means are diverging as π increases. However, since agents can switch randomly from one type to the other, it is difficult to fully characterize the equilibrium without resorting to numerical simulations.

7.5. More on directed search: the case with no repos

In the context of a lending relationship, we usually understand "counterparty risk" as the risk that a borrower fails to repay his debt. In the paper, we illustrate another type of

[21]This is a usual problem in models with pairwise trade and arbitrary asset holdings. Agents in Kiyotaki and Wright (1989) or Duffie et. al. (2005) trade an indivisible asset with a unit upper bound. Lagos and Wright (2005) introduces a Walrasian market with quasi-linear preferences so that agents can level their asset holdings, thus giving a degenerate distribution of assets. In a Lagos-Wright environment, there is no role for repos.

counterparty risk: The risk that the next counterparty does not hold the righ amount of the asset. To show this, we re-consider our simple two-period example. In this section, we assume that, in addition to being matched with the right type (h or ℓ), an agent holding a is always being matched with an agent holding $2A-a$. To make sure that this is always feasible, we have to modify the environment slightly by adding "noise traders". These traders do not hold the equilibrium portofolio and an agent who deviates from the equilibrium strategy and holds $a \neq a^*$ can always be matched with a noise trader holding $2A-a$. This is how this example differs from our earlier one.

Under this assumption the first order conditions on the Walrasian market at $t=1$ are still given by (4) and (5). Prices still satisfy $p^s = p^r + \beta V_i'(a_i^*)$ and $p^r = u_i'(c_i^*)$. Moreover, the value of holding a for a type i at the start of the bargaining game is still given by $v_i(a)$ for $i = h, \ell$ as described in (6) and (7) with the added requirement that $a_{-i} = 2A - a_i$, or

$$v_h(a) = u_h(a) + \theta S(a, 2A-a) \quad (16)$$

$$v_\ell(a) = u_\ell(a) + (1-\theta) S(2A-a, a) \quad (17)$$

where $S(a_h, a_\ell) = u_h(c_h^*) + u_\ell(c_\ell^*) - u_h(a_h) - u_\ell(a_\ell)$ is again the match surplus from trade. Notice that, contrary to our earlier example, an agent's payoff only depends on his asset holdings, as in a Walrasian market. Loosely speaking, agents now have better "control" over the optimum level of assets. Notice that with the new matching technology, $c_h + c_\ell = 2A$ so that $dc_h/da_h = -dc_\ell/da_h$. We can then compute the marginal expected value for agent i of holding asset a_i at date 2 as,

$$V_i'(a) = \pi \left\{(1-\theta_i)u_i'(a) + \theta_i u_{-i}'(2A-a)\right\} + (1-\pi)\left\{(1-\theta_{-i})u_{-i}'(a) + \theta_{-i} u_i'(2A-a)\right\}$$

where $\theta_h = \theta$ and $\theta_\ell = 1-\theta$. Hence, evaluating at $a = c_h^*$ and c_ℓ^* for V_h' and V_ℓ' respectively, we obtain

$$V_h'(c_h^*) = \pi \left\{(1-\theta)u_h'(c_h^*) + \theta u_\ell'(c_\ell^*)\right\} + (1-\pi)\left\{\theta u_\ell'(c_h^*) + (1-\theta)u_h'(c_\ell^*)\right\}$$

and

$$V_\ell'(c_\ell^*) = \pi \left\{\theta u_\ell'(c_\ell^*) + (1-\theta)u_h'(c_h^*)\right\} + (1-\pi)\left\{(1-\theta)u_h'(c_\ell^*) + \theta u_\ell'(c_h^*)\right\}$$

so that $V_\ell'(c_\ell^*) = V_h'(c_h^*)$. Since $u_h'(c_h^*) = u_\ell'(c_\ell^*)$ there is no wedge at (c_h^*, c_ℓ^*) between the value of present and future consumption. As a consequence there is no repo and only the sales market is active at $t = 1$.

7.6. Transaction wedge

Duffie (1996) introduces transaction costs to model the need for special repo. It is indeed fair to argue that repo may be preferred if they are treated differently from a fiscal perspective, for example. Therefore, in this subsection, we study the robustness of our results when there is a wedge – positive or negative – between the transactions costs of the repo and sale.

More specifically, we assume that agents are matched bilaterally in two markets: a sale and a repo market. They first meet in the sale market and trade their asset permanently in bilateral matches. In the repo market, agents borrow/lend assets bilaterally for one period. In both markets, agents use the numeraire for settlement. However, there is a wedge between the production cost and consumption benefit when settling the terms of a sale, while there is no wedge when agents settle a repo. More specifically, when a type h agent with asset holding a_h is matched with a type ℓ agent with asset holding a_ℓ in the sale market they solve the following problem,

$$\max_{d^s, q^s} \quad \left[W_h^r\left(a_h + q^s\right) - \omega \cdot d^s - W_h^r\left(a_h\right)\right]^{\theta_s} \left[W_\ell^r\left(a_\ell - q^s\right) + d^s - W_\ell^r\left(a_\ell\right)\right]^{1-\theta_s}, \tag{18}$$

where $W_i^r(a)$ is the value of going to the repo market with asset holding a for a type $i \in \{h, \ell\}$ agent. In the repo market, a type h agent with asset holding a_h' meets a type ℓ agent with asset holding a_ℓ', and they solve the following problem,

$$\max_{d^r, q^r} \quad \left[u_h\left(a_h' + q^r\right) - d^r - u_h\left(a_h'\right)\right]^{\theta_r} \left[u_\ell\left(a_\ell' - q^r\right) + d^r - u_\ell\left(a_\ell'\right)\right]^{1-\theta_r}. \tag{19}$$

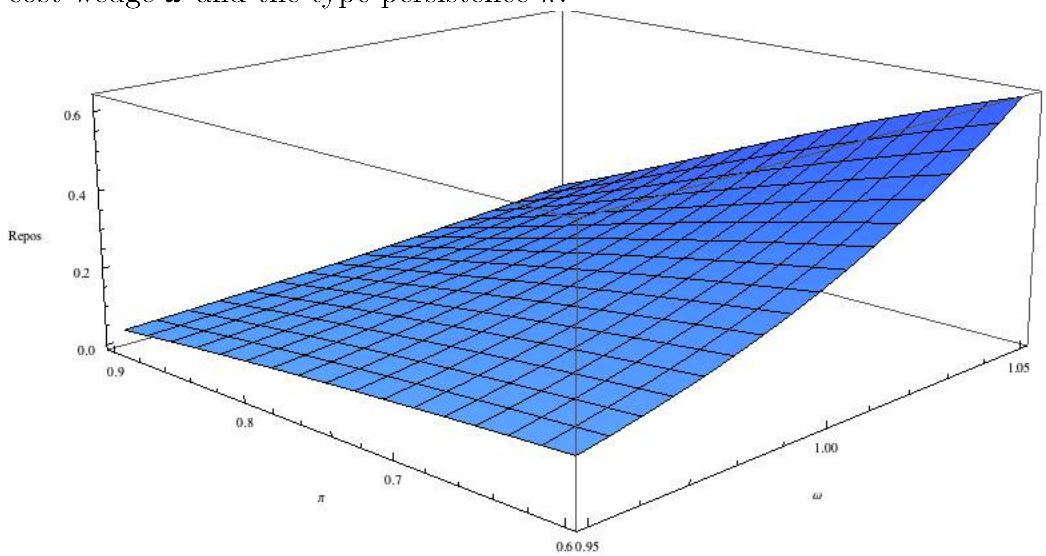
Note that we are also allowing for different bargaining powers in the repo and sale markets. The reason is that we are not necessarily assuming that agents are trading with the same counterparty in both markets. Since agents can only adjust their asset holdings temporarily in the repo market, we have

$$\begin{aligned} W_h^r\left(a_h'\right) &= u_h\left(a_h' + q^r\right) - d^r + \beta\left\{\pi \cdot W_h^s\left(a_h'\right) + (1-\pi) \cdot W_\ell^s\left(a_h'\right)\right\} \\ W_\ell^r\left(a_\ell'\right) &= u_\ell\left(a_\ell' - q^r\right) + d^r + \beta\left\{\pi \cdot W_\ell^s\left(a_\ell'\right) + (1-\pi) \cdot W_h^s\left(a_\ell'\right)\right\} \end{aligned}$$

where

$$W_h^s(a_h) = W_h^r(a_h + q^s) - \omega \cdot d^s$$
$$W_\ell^s(a_\ell) = W_\ell^r(a_\ell - q^s) + d^s.$$

In the equilibrium we have $W_h^{r\prime}(a_h^*) = \omega \cdot W_\ell^{r\prime}(a_\ell^*)$. In the Appendix, we use this equation as well as $a_\ell^* = 2A - a_h^*$ to solve for a_h^* and a_ℓ^*. We find that an increase in the wedge ω decreases a_h^* and increases a_ℓ^*. In other words, when the readjusting asset holding via sale becomes more costly, more adjustment is done via repos. The following figure shows a numerical example of how the share of repos in readjustment changes with respect to changes in the sale cost wedge ω and the type persistence π.



Note that as the sale cost wedge ω rises, the share of repos in the asset readjustments increases up to a level that all of the readjustments are done via repos. The opposite holds as well. If sale becomes much cheaper than repo, then all readjustments would be conducted via sale market. For the intermediate levels of the wedge ω, both of sale and repo coexists.

7.7. Ex-ante Asymmetric Agents

It is common for some participants of the repos market to be always on one side of the market, participating only as "lenders" or as "borrowers". In this subsection we study a case of ex-ante asymmetric agents to show that repo is essential for achieving efficient allocation even if agents always participate on one side of the market. More specifically, assume there are equal measure of two types of agents denoted by B and L. Each type receives a nonpersistent i.i.d preference shock h or ℓ, such that in every period π fraction of type B agents have h preference shock and π fraction of type L agents have ℓ preference shock. Denoting the utility from asset holding of type B agents with h and ℓ preferences by $u_{B,h}(\cdot)$ and $u_{B,\ell}(\cdot)$, and for type L agents with h and ℓ preferences by $u_{L,h}(\cdot)$ and $u_{L,\ell}(\cdot)$, we assume that $u'_{B,h}(a) > u'_{B,\ell}(a) > u'_{L,h}(a) > u'_{L,\ell}(a)$. Moreover, the efficient allocation is described by the consumption levels $c^*_{B,h}, c^*_{B,\ell}, c^*_{L,h}$ and $c^*_{L,\ell}$ that satisfy

$$u'_{B,h}(c^*_{B,h}) = u'_{B,\ell}(c^*_{B,\ell}) \quad = \quad u'_{L,h}(c^*_{L,h}) = u'_{L,\ell}(c^*_{L,\ell}), \tag{20}$$

for

$$\begin{aligned} 2A &= c^*_{B,h} + c^*_{L,\ell} \\ &= c^*_{B,\ell} + c^*_{L,h}, \end{aligned}$$

where A is the average asset endowment in the economy. We assume that agents with opposite types meet bilaterally via directed search after realization of their shocks in every period and receive their utility from asset after adjusting their portfolios. Terms of trade are set with bargaining power θ and $1-\theta$ for type B and L. In the Appendix we show that asset sales alone cannot achieve the efficient allocation,

Proposition 10. *With two types of agents who are subject to preference shock, only asset sales in bilateral trade cannot implement the efficient allocation, even as the preference shock disappears.*

Finally, it is straightforward to show that given (20), there exist an efficient equilibrium using repo transactions. In particular, if all type B agents hold $c^*_{B,\ell}$ units and type L agents hold $c^*_{L,h}$ units of asset, then by repo of $c^*_{B,h} - c^*_{B,\ell} = c^*_{L,\ell} - c^*_{L,h}$ units of asset between type B agents with h shock and type L agents with ℓ shock, an efficient allocation is attained as an equilibrium outcome. Note that in this equilibrium type B agents only participate on the borrower side of the repo market and type L agents only participate on the lender side.

8. Conclusion

This paper presents a simple environment with trading frictions where agents trade both in repo and asset markets. Repos are useful because agents can enjoy the service from holding the asset today, without changing the composition of their future portfolio. We find this is important for agents to maintain an appropriate outside option in bilateral trade: Holding too few assets (selling too much) would weaken the agent's future position, while holding too many (selling too few) would diminish the marginal value of the asset, which would then be relinquished "at a low price." The friction giving rise to repo is bilateral trade: Then, contrary to Walrasian trading, the terms of trade depend on both agents' asset holdings. Therefore any other friction or market structure implying that the realized value of the asset depends on one's asset holding, and where the expected future value of the asset differs from the actual realized value of the asset should deliver a similar result.

With directed matching and two valuations, we characterize an equilibrium as a two-point supports. These two equilibrium levels of asset holding are converging as the valuation shock becomes more persistent. We find that the volume for repos is always decreasing in the persistence of the valuation shock, while the volume of asset sales is hump-shaped. This hump is explained by two interacting margins: On the one hand, less agents are switching valuation when it becomes more persistent, but on the other hand they trade a larger quantity each time they switch valuation in order to hold the equilibrium amount.

This has interesting implications for the organization of the repo market. In particular our theory predicts that the repos market will be thinner when there is little uncertainty about one's future preferences. Although we leave it for future research, we suspect that monetary policy (which is operated in the repo market) will have a higher impact then, as a lower quantity of repos can affect the market. Similarly, starting from a situation where agents know their future preferences, as uncertainty is growing, so is the volume of asset sales. Therefore, more sales have to be conducted in order to move the market. If we associate "normal times" with times when agents have a good idea about their future preferences, then monetary policy should be conducted with repos. However, with uncertainty growing overly large, monetary policy will be more effective in moving markets price if it is conducted via asset sales/purchases.

9. Appendix

Proof of Proposition 2

We show that no two agents have a benefit of being matched with each other than their prescribed partner. It should be clear that the bargaining solution is always better than autarky (although not in a strict sense). Therefore we only need to check deviations by a coalition of 2 agents. An agent ℓ with $\bar{a}_h$ could decide to form a coalition with an agent ℓ with $\bar{a}_\ell$ or an agent h with $\bar{a}_h$. It is a property of the bargaining solution that an agent ℓ will obtain a lower payoff being matched with an agent h with a higher amount of asset. Indeed, the agent ℓ can extract less from a relatively rich agent h, as the marginal utility of obtaining more of the asset is lower for this agent. Hence, an agent ℓ with $\bar{a}_h$ prefers to be matched with an agent h with $\bar{a}_\ell$. Also, it is a property of the bargaining solution that, given he has to meet an agent holding a, an agent ℓ prefers to be matched with the agent with the highest marginal utility and so an agent h.

We now turn to agents h. An agent h with $\bar{a}_\ell$ could decide to form a coalition with an agent h with $\bar{a}_h$ or an agent ℓ with $\bar{a}_\ell$. As above, however, it is a property of the bargaining solution that the payoff of agent h matched with an agent ℓ is higher whenever the agent ℓ is holding more asset. Hence, the agent h will not want to be matched with an agent ℓ holding $\bar{a}_\ell$. Also, an agent h with $\bar{a}_h$ prefers to be matched with the agent holding $\bar{a}_\ell$ with the lowest marginal utility, i.e. with an ℓ agent. Therefore there is no 2-agents coalition where both agents would do better than under the prescribed matching technology, which shows that, combined with the distribution over $\{\bar{a}_h, \bar{a}_\ell\}$, it is an equilibrium.

Proof of Proposition 3

The value functions are

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)] \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)] \end{aligned}$$

Adding both equations, we obtain

$$V_h(\bar{a}_h) + V_\ell(\bar{a}_\ell) = \frac{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r)}{1-\beta} \tag{21}$$

Also

$$\begin{aligned}
V_h(\bar{a}_\ell) &= \pi[u_h(\bar{a}_h+q^r)-d(\bar{a}_\ell,\bar{a}_h)+\beta V_h(\bar{a}_h)]+(1-\pi)[u_\ell(\bar{a}_\ell-q^r)+d(\bar{a}_h,\bar{a}_\ell)+\beta V_\ell(\bar{a}_\ell)] \\
V_\ell(\bar{a}_h) &= \pi[u_\ell(\bar{a}_\ell-q^r)+d(\bar{a}_\ell,\bar{a}_h)+\beta V_\ell(\bar{a}_\ell)]+(1-\pi)[u_h(\bar{a}_h+q^r)-d(\bar{a}_h,\bar{a}_\ell)+\beta V_h(\bar{a}_h)]
\end{aligned}$$

and adding both equations, we obtain also

$$V_h(\bar{a}_\ell)+V_\ell(\bar{a}_h)=V_h(\bar{a}_h)+V_\ell(\bar{a}_\ell) \tag{22}$$

From the first order conditions of the bargaining problem, we then can compute $d(\bar{a}_h,\bar{a}_\ell)$ as

$$d(\bar{a}_h,\bar{a}_\ell) = (1-\theta)[u_h(\bar{a}_h+q^r)-u_h(\bar{a}_h)]-\theta[u_\ell(\bar{a}_\ell-q^r)-u_\ell(\bar{a}_\ell)] \tag{23}$$

where we have used (21)-(22) and the fact that $q^s(\bar{a}_h,\bar{a}_\ell)=0$. Therefore, using (23) we obtain

$$\begin{aligned}
u_h(\bar{a}_h+q^r)-d(\bar{a}_h,\bar{a}_\ell)+\beta V_h(\bar{a}_h) &= \\
u_h(\bar{a}_h)+\beta V_h(\bar{a}_h)+\theta[u_h(\bar{a}_h+q^r)+u_\ell(\bar{a}_\ell-q^r)-u_h(\bar{a}_h)-u_\ell(\bar{a}_\ell)]
\end{aligned}$$

Also

$$\begin{aligned}
u_\ell(\bar{a}_\ell-q^r)+d(\bar{a}_h,\bar{a}_\ell)+\beta V_\ell(\bar{a}_\ell) &= \\
u_\ell(\bar{a}_\ell)+\beta V_\ell(\bar{a}_\ell)+(1-\theta)[u_h(\bar{a}_h+q^r)+u_\ell(\bar{a}_\ell-q^r)-u_h(\bar{a}_h)-u_\ell(\bar{a}_\ell)]
\end{aligned}$$

In a similar fashion, using $q^s(\bar{a}_\ell,\bar{a}_h)=\bar{a}_h-\bar{a}_\ell$ we can rewrite $d(\bar{a}_\ell,\bar{a}_h)$ as

$$d(\bar{a}_\ell,\bar{a}_h) = d(\bar{a}_h,\bar{a}_\ell)+\bar{u}+\beta(1-\theta)[V_h(\bar{a}_h)-V_h(\bar{a}_\ell)]+\beta\theta[V_\ell(\bar{a}_h)-V_\ell(\bar{a}_\ell)] \tag{24}$$

where

$$\bar{u}=(1-\theta)[u_h(\bar{a}_h)-u_h(\bar{a}_\ell)]+\theta[u_\ell(\bar{a}_h)-u_\ell(\bar{a}_\ell)]$$

Therefore, using (22) and (24) and simplifying we obtain

$$\begin{aligned}
u_\ell(\bar{a}_\ell-q^r)+d(\bar{a}_\ell,\bar{a}_h)+\beta V_\ell(\bar{a}_\ell) &= \\
u_\ell(\bar{a}_h)+\beta V_\ell(\bar{a}_h)+(1-\theta)[u_h(\bar{a}_h+q^r)+u_\ell(\bar{a}_\ell-q^r)-u_h(\bar{a}_\ell)-u_\ell(\bar{a}_h)]
\end{aligned}$$

Similarly

$$
\begin{aligned}
u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h) &= \\
u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_\ell) + \theta[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)]
\end{aligned}
$$

Hence, combining all these expressions, we obtain

$$
\begin{aligned}
V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h) + \beta V_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{a}_h) + \beta V_\ell(\bar{a}_h)] + \pi\theta S + (1-\pi)(1-\theta)\tilde{S} \\
V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_\ell)] + \pi(1-\theta)S + (1-\pi)\theta\tilde{S} \\
V_h(\bar{a}_\ell) &= \pi[u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_\ell)] + (1-\pi)[u_\ell(\bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)(1-\theta)S + \pi\theta\tilde{S} \\
V_\ell(\bar{a}_h) &= \pi[u_\ell(\bar{a}_h) + \beta V_\ell(\bar{a}_h)] + (1-\pi)[u_h(\bar{a}_h) + \beta V_h(\bar{a}_h)] + (1-\pi)\theta S + \pi(1-\theta)\tilde{S}
\end{aligned}
$$

where

$$
\begin{aligned}
S &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_h) - u_\ell(\bar{a}_\ell) \\
\tilde{S} &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)
\end{aligned}
$$

Solving for $V_h(\bar{a}_h)$ we obtain

$$
(1-\beta)V_h(\bar{a}_h) = \frac{(1-\pi)[u_\ell(\bar{a}_h) + (1-\theta)\tilde{S}] + [\pi - (2\pi-1)\beta][u_h(\bar{a}_h) + \theta S]}{1-(2\pi-1)\beta}
$$

And taking the derivative, we have

$$
(1-\beta)V_h'(\bar{a}_h) = \frac{u_\ell'(\bar{a}_h)(1-\pi) + [\pi - (2\pi-1)\beta]u_h'(\bar{a}_h) + (1-\pi)(1-\theta)\frac{\partial \tilde{S}}{\partial \bar{a}_h} + \theta\frac{\partial S}{\partial \bar{a}_h}[\pi - (2\pi-1)\beta]}{1-(2\pi-1)\beta}
$$

Using the first order condition for q^r we obtain after some simplifications,

$$
\begin{aligned}
(1-\beta)(1-(2\pi-1)\beta)V_h'(\bar{a}_h) &= u_\ell'(\bar{a}_h)\theta(1-\pi) + u_h'(\bar{a}_h)(1-\theta)[\pi - (2\pi-1)\beta] \\
&\quad + u_h'(\bar{a}_h + q^r)[1-\pi + (2\pi-1)(1-\beta)\theta]
\end{aligned}
$$

Since (21) holds, we use the first order condition for q^r and simplify to obtain

$$
\begin{aligned}
(1-\beta)(1-(2\pi-1)\beta)V_\ell'(\bar{a}_\ell) &= u_\ell'(\bar{a}_\ell - q^r)[\pi - (2\pi-1)(\beta + (1-\beta)\theta)] + (1-\pi)(1-\theta)u_h'(\bar{a}_\ell) \\
&\quad + \theta[\pi - (2\pi-1)\beta]u_\ell'(\bar{a}_\ell)
\end{aligned}
$$

The first condition for q^s imposes that $V_h'(\bar{a}_h) = V_\ell'(\bar{a}_\ell)$. Using the fact that $u_\ell'(\bar{a}_\ell - q^r) = u_h'(\bar{a}_h + q^r)$ and simplifying, we obtain

$$u_h'(\bar{a}_h + q^r) \quad = \quad \frac{[\pi - (2\pi-1)\beta][\theta u_\ell'(\bar{a}_\ell) - (1-\theta)u_h'(\bar{a}_h)] - (1-\pi)[\theta u_\ell'(\bar{a}_h) - (1-\theta)u_h'(\bar{a}_\ell)]}{(2\pi-1)(1-\beta)(2\theta-1)}$$

Together with the first order condition on asset sales and the feasibility constraint, this completes the proof.

Proof of Corollary 4

The equilibrium allocation is given by

$$\begin{aligned} u_h'(\bar{c}_h) &= u_\ell'(\bar{c}_\ell) \\ u_h'(\bar{c}_h) &= \frac{[\pi - (2\pi-1)\beta][\theta u_\ell'(\bar{a}_\ell) - (1-\theta)u_h'(\bar{a}_h)] - (1-\pi)[\theta u_\ell'(\bar{a}_h) - (1-\theta)u_h'(\bar{a}_\ell)]}{(2\pi-1)(1-\beta)(2\theta-1)} \\ \bar{c}_h + \bar{c}_\ell &= \bar{a}_h + \bar{a}_\ell = 2A \end{aligned} \tag{25}$$

Let

$$\alpha_1(\pi) = \frac{\pi - (2\pi-1)\beta}{2\pi-1} = \frac{\pi}{2\pi-1} - \beta$$

and

$$\alpha_2(\pi) = \frac{1-\pi}{2\pi-1}$$

then $\alpha_1'(\pi) = \alpha_2'(\pi) = \frac{-1}{(2\pi-1)^2} < 0$. And we can rewrite (25) as

$$u_h'(\bar{c}_h)(1-\beta)(2\theta-1) = \alpha_1(\pi)[\theta u_\ell'(\bar{a}_\ell) - (1-\theta)u_h'(\bar{a}_h)] - \alpha_2(\pi)[\theta u_\ell'(\bar{a}_h) - (1-\theta)u_h'(\bar{a}_\ell)]$$

Notice that $\bar{c}_h$ is not a function of π, so that using $\bar{a}_h + \bar{a}_\ell = A$ and the implicit function theorem, we have

$$\begin{aligned} 0 \quad = \quad & \alpha_1'(\pi)[\theta u_\ell'(\bar{a}_\ell) - (1-\theta)u_h'(\bar{a}_h)]d\pi + \alpha_1(\pi)[-\theta u_\ell''(A - \bar{a}_h) - (1-\theta)u_h''(\bar{a}_h)]d\bar{a}_h && (26) \\ & -\alpha_2'(\pi)[\theta u_\ell'(\bar{a}_h) - (1-\theta)u_h'(\bar{a}_\ell)]d\pi - \alpha_2(\pi)[\theta u_\ell''(\bar{a}_h) + (1-\theta)u_h''(A - \bar{a}_h)]d\bar{a}_\ell && (27) \end{aligned}$$

which we can simplify as

$$\frac{d\bar{a}_h}{d\pi} \quad = \quad \frac{\alpha_1'(\pi)\left\{\theta\left[u_\ell'(\bar{a}_\ell) - u_\ell'(\bar{a}_h)\right] + (1-\theta)\left[u_h'(\bar{a}_\ell) - u_h'(\bar{a}_h)\right]\right\}}{\alpha_1(\pi)[\theta u_\ell''(A - \bar{a}_h) + (1-\theta)u_h''(\bar{a}_h)] + \alpha_2(\pi)[\theta u_\ell''(\bar{a}_h) + (1-\theta)u_h''(A - \bar{a}_h)]} \tag{28}$$

Since $u_i'(\bar{a}_\ell) > u_i'(\bar{a}_h)$ for both i and $\alpha_i'(\pi) < 0$, the numerator is negative. Concavity of the utility function implies that the denominator is also negative. Therefore we have $d\bar{a}_h/d\pi > 0$.

Given π the volume of repo in this economy is given by q^r (since all agents use repo) while the volume of asset sales is given by $(1-\pi)q^s = (1-\pi)(\bar{a}_h - \bar{a}_\ell)$. Clearly, the sales volume is hump shaped as when $\pi = 1/2$ we have $\bar{a}_h = \bar{a}_\ell$ so that $q^s = 0$ while when $\pi = 1$, $q^s = 0$ as well. However, $(1-\pi)q^s > 0$ for all other values of π. Since the problem is continuous, sales volume is hum-shaped. Also, the fact that $\bar{c}_h = \bar{a}_h + q^r$ is a constant implies that total volume of repo (i.e. q^r since all agents engage in repo) is declining in π. Since there are no repo when $\pi = 1$, the volume of repo is declining to zero.

10. Appendix for Online Publication

Example

This example seeks to illustrate the role of pairwise matching *today* to explain the usefulness of repos. It also indicates how linear utilities from holding the asset affects the results. We suppose that an agent $i = h, \ell$ derives utility from consuming fruits in both periods according to the utility $U_i(c_1, c_2) = u_i(c_1) + \beta\lambda_i c_2$, where $u_i(c)$ satisfies Assumption 1 while $\lambda_h \geq \lambda_\ell$, c_1 is the quantity of fruits consumed at $t = 1$ and c_2 is the quantity consumed at $t = 2$. We assume $\pi = 1$ so that agents do not switch type. An agent h is matched with an agent ℓ and they bargain over the allocation. There is no further trade in period 2. We assume that $a_h + a_\ell = 2A$. Given there is no trade at $t = 2$, the bargaining problem at $t = 1$ between agent h with bargaining power $\theta \in [0, 1]$ and agent ℓ with bargaining power $1 - \theta$ is

$$\max_{c_h, c_\ell, d} \left[u_h(c_h) - d + \beta\lambda_h q^s - u_h(a_h)\right]^\theta \left[u_\ell(c_\ell) + d - \beta\lambda_\ell q^s - u_\ell(a_\ell)\right]^{1-\theta}$$

subject to $c_h + c_\ell = a_h + a_\ell$ and $q^s \in [-a_h, a_\ell]$. The solution to the bargaining problem is then

$$\begin{aligned} u_h'(c_h^*) &= u_\ell'(c_\ell^*) \\ (a_\ell - q^s)(\lambda_h - \lambda_\ell) &= 0 \\ d(q^s, q^r) &= (1-\theta)[u_h(c_h^*) + \beta\lambda_h q^s - u_h(a_h)] + \theta[u_\ell(c_\ell^*) - \beta\lambda_\ell q^s - u_\ell(a_\ell)] \end{aligned} \quad (29)$$

where $c_h^* = a_h + q^r + q^s$ and $c_\ell^* = a_\ell - q^r - q^s$. Clearly, Assumption 1 implies $c_h^* > c_\ell^*$. Also these optimal consumption levels are uniquely pinned down by the first order conditions together with $c_h^* + c_\ell^* = a_h + a_\ell$. Agents just choose the mix between repo and asset sales to achieve those levels. If $\lambda_h > \lambda_\ell$ then the solution is $q^s = a_\ell$, and $q^r = -c_\ell^*$. In words, agent ℓ sells all his assets to agent h but repo some to achieve the desired level of consumption at $t = 1$.

Under which conditions would agent ℓ be indifferent between repo and asset sales? To be indifferent, agent ℓ should obtain the same payoff by keeping some amount $\tilde{a}_\ell > 0$ into period 2 instead of selling everything at $t = 1$. If he keeps $\tilde{a}_\ell$, he can lower the amount of security he rents to $\tilde{q}^r = -c_\ell^* + \tilde{a}_\ell$, while he only sells $\tilde{q}^s = a_\ell - \tilde{a}_\ell$. In turn, his new total transfer is $d(a_\ell - \tilde{a}_\ell, \tilde{q}^r)$ instead of $d(a_\ell, -c_\ell^*)$. Therefore agent ℓ would be indifferent keeping some assets or selling it all if and only if

$$u(c_\ell^*) + d(a_\ell, -c_\ell^*) = u(c_\ell^*) + d(a_\ell - \tilde{a}_\ell, \tilde{q}^r) + \beta\lambda_\ell\tilde{a}_\ell$$

where the last term on the right hand side is the payoff from carrying over asset into the next period. This expression can be simplified to

$$d(a_\ell, -c_\ell^*) - d(a_\ell - \tilde{a}_\ell, \tilde{q}^r) = \beta\lambda_\ell\tilde{a}_\ell.$$

However, using (29) we obtain that

$$d(a_\ell, -c_\ell^*) - d(a_\ell - \tilde{a}_\ell, \tilde{q}^r) = \theta\beta\lambda_\ell\tilde{a}_\ell + (1-\theta)\beta\lambda_h\tilde{a}_\ell \geq \beta\lambda_\ell\tilde{a}_\ell$$

In words, agent ℓ prefers to sell his entire portfolio to agent h today as he is able to extract some of agent h's higher valuation of the asset tomorrow for his own benefit today (which is a similar effect to Lagos and Rocheteau). Notice that if $\lambda_h = \lambda_\ell$ then both agents would have the same valuation for the asset at $t = 2$ and agent ℓ would be indifferent between repo and asset sales at $t = 1$, as in the Walrasian benchmark. Also, if $\theta = 1$ then agent ℓ would be unable to extract some of the future gain from agent h and so agent ℓ would be indifferent between repo and asset sales. In all other cases, i.e. whenever $\lambda_h > \lambda_\ell$ and $\theta < 1$, repos are useful to equate the marginal utilities at $t = 1$ and to achieve efficiency. In the general model, we endogenize the marginal utility λ_h and λ_ℓ, but the intuition remains the same.

Proof of Proposition 6

Given the time to the next meeting is $\Delta < 1$, we denote the asset holdings and repo level by $\bar{a}_{\ell,\Delta}$, $\bar{a}_{h,\Delta}$ and q^r_Δ. Using this notation, we have $c_\ell^* = \bar{a}_\ell - q^r = \bar{a}_{\ell,\Delta} - q^r_\Delta$ and $c_h^* = \bar{a}_h + q^r = \bar{a}_{h,\Delta} + q^r_\Delta$. Using the equilibrium condition of Proposition 3, we get

$$\begin{aligned}
&(2\theta-1)u_h'(c_h^*) \\
=\ & \frac{[\pi - (2\pi-1)\beta][\theta u_\ell'(c_\ell^* + q^r) - (1-\theta)u_h'(c_h^* - q^r)] - (1-\pi)[\theta u_\ell'(c_h^* - q^r) - (1-\theta)u_h'(c_\ell^* + q^r)]}{(2\pi-1)(1-\beta)} \\
=\ & \frac{[\pi_\Delta - (2\pi_\Delta - 1)\beta_\Delta][\theta u_\ell'(c_\ell^* + q^r_\Delta)) - (1-\theta)u_h'(c_h^* - q^r_\Delta)]}{(2\pi_\Delta - 1)(1-\beta_\Delta)} \\
& - \frac{(1-\pi_\Delta)[\theta u_\ell'(c_h^* - q^r_\Delta) - (1-\theta)u_h'(c_\ell^* + q^r_\Delta))]}{(2\pi_\Delta - 1)(1-\beta_\Delta)}
\end{aligned}$$

which is equivalent to

$$\begin{aligned} & LH(q^r) + \left(\frac{(1-\pi)}{(1-2(1-\pi))(1-\beta)} \right) (LH(q^r) - RH(q^r)) \\ = \;& LH(q^r_\Delta) + \left(\frac{(1-\pi_\Delta)}{(1-2(1-\pi_\Delta))(1-\beta_\Delta)} \right) (LH(q^r_\Delta) - RH(q^r_\Delta)) \end{aligned} \tag{30}$$

where

$$LH(q^r) = \theta u'_\ell(c^*_\ell + q^r) - (1-\theta) u'_h(c^*_h - q^r)$$

and

$$RH(q^r) = \theta u'_\ell(c^*_h - q^r) - (1-\theta) u'_h(c^*_\ell + q^r).$$

For $q \leq (c_h{}^* - c_\ell{}^*)/2$, we have

$$u'_\ell(c^*_h - q) < u'_\ell(c^*_\ell + q) < u'_\ell(c^*_\ell) = u'_h(c^*_h) < u'_h(c^*_h - q) < u'_h(c^*_\ell + q)$$

therefore we have $LH(q) > RH(q)$. Moreover, concavity of u_h and u_ℓ implies

$$\frac{d}{dq} LH(q) < 0 < \frac{d}{dq} RH(q).$$

Now, notice that for $\Delta < 1$

$$\begin{aligned} \frac{(1-\pi_\Delta)}{(1-2(1-\pi_\Delta))(1-\beta_\Delta)} &= \frac{(1-\pi)}{(1-2\Delta(1-\pi))(1-\beta)} \\ &< \frac{(1-\pi)}{(1-2(1-\pi))(1-\beta)} \end{aligned}$$

therefore (30) implies $q^r_\Delta < q^r$ for $\Delta < 1$. Notice that as $\Delta \to 0$, the share of reallocation via repo decreases to $q^r_0 > 0$ which is determined by

$$(2\theta - 1) u'_h(c^*_h) = LH(q^r_0) + \frac{(1-\pi)}{(1-\beta)} (LH(q^r_0) - RH(q^r_0)).$$

This completes the proof.

Proof of Proposition 7

We still assume that agents who did not switch are matched together, while those agents who just switched are matched with each other. With Nash bargaining, the allocation of an agent h with portfolio a_h matched with an agent ℓ with portfolio a_ℓ solves the following problem:

$$\begin{aligned}\max_{q^s,q^r,d}&[u_h(a_h+q^s+q^r)-d+\beta V_h(a_h+q^s)-u_h(a_h)-\beta\tilde{W}_h(a_h)]^\theta\\ &\times[u_\ell(a_\ell-q^s-q^r)+d+\beta V_\ell(a_\ell-q^s)-u_\ell(a_\ell)-\beta\tilde{W}_\ell(a_\ell)]^{1-\theta}\end{aligned}$$

with first order conditions

$$\begin{aligned}V_h'(a_h+q^s) &= V_\ell'(a_\ell-q^s)\\ u_h'(a_h+q^s+q^r) &= u_\ell'(a_h-q^s-q^r)\\ d(a_h,a_\ell) &= (1-\theta)[u_h(a_h+q^s+q^r)+\beta V_h(a_h+q^s)-u_h(a_h)-\beta\tilde{W}_h(a_h)]\\ &\quad -\theta[u_\ell(a_\ell-q^s-q^r)+\beta V_\ell(a_\ell-q^s)-u_\ell(a_\ell)-\beta\tilde{W}_\ell(a_\ell)]\end{aligned}$$

We still assume that agents who did not switch types are matched together while those agents who just switched are matched together. We first solve for $\tilde{W}_i(a)$. By definition, $\tilde{W}_i(a)=\pi W_i(a)+(1-\pi)W_j(a)$ with $i\neq j\in\{h,\ell\}$ and where W_i denotes the value of participating in the Walrasian market as a type i. From the problem of agents in the Walrasian market, it should be clear that $W_i(a)=pa+W_i(0)$, where $W_i(0)$ is given by

$$\begin{aligned}W_i(0) &= u_i(a_i^w)-pa_i^w+\beta E_{k|i}W_k(a_i^w)\\ &= u_i(a_i^w)-p^r a_i^w+\beta E_{k|i}W_k(0)\end{aligned}$$

where a_i^w is the solution to $u_i'(a_i^w)=p^r$ and $u_h'(a_h^w)=u_\ell'(a_\ell^w)$ with $a_\ell^w+a_h^w=2A$. Solving for $W_i(0)$ we have

$$\begin{aligned}W_h(0) &= u_h(a_h^w)-p^r a_h^w+\beta\pi W_h(0)+\beta(1-\pi)W_\ell(0)\\ W_\ell(0) &= u_\ell(a_\ell^w)-p^r a_\ell^w+\beta\pi W_\ell(0)+\beta(1-\pi)W_h(0)\end{aligned}$$

so that

$$(1-\beta)W_h(0)=\alpha[u_h(a_h^w)-p^r a_h^w]+(1-\alpha)[u_\ell(a_\ell^w)-p^r a_\ell^w]$$

where $\alpha = \frac{1-\beta\pi}{1+\beta-2\beta\pi} \in [0,1]$. Similarly,

$$(1-\beta)W_\ell(0) = \alpha[u_\ell(a_\ell^w) - p^r a_\ell^w] + (1-\alpha)[u_h(a_h^w) - p^r a_h^w]$$

Therefore,

$$\begin{aligned}(1-\beta)\tilde{W}_h(0) &= \pi(1-\beta)W_h(0) + (1-\pi)(1-\beta)W_\ell(0) \\ &= u(a_h^w) - p^r a_h^w + [\pi + \alpha - 2\pi\alpha][u_\ell(a_\ell^w) - p^r a_\ell^w - u_h(a_h^w) + p^r a_h^w]\end{aligned}$$

and

$$\begin{aligned}(1-\beta)\tilde{W}_\ell(0) &= (1-\pi)(1-\beta)W_h(0) + \pi(1-\beta)W_\ell(0) \\ &= u(a_\ell^w) - p^r a_\ell^w + [\pi + \alpha - 2\pi\alpha][u_h(a_h^w) - p^r a_h^w - u_\ell(a_\ell^w) + p^r a_\ell^w]\end{aligned}$$

Notice that

$$\tilde{W}_h(0) + \tilde{W}_\ell(0) = \frac{u(a_h^w) - p^r a_h^w + u(a_\ell^w) - p^r a_\ell^w}{1-\beta}$$

In this environment the first order condition of the bargaining problem gives us

$$u_h'(\bar{a}_h + q^r) = u_\ell'(\bar{a}_\ell - q^r)$$

so that

$$\begin{aligned}\bar{a}_h + q^r &= a_h^w, \\ \bar{a}_\ell - q^r &= a_\ell^w.\end{aligned}$$

The value functions are

$$\begin{aligned}V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)] \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)]\end{aligned}$$

Adding both equations, we obtain

$$V_h(\bar{a}_h) + V_\ell(\bar{a}_\ell) = \frac{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r)}{1-\beta} = \tilde{W}_h(a_h^w) + \tilde{W}_\ell(a_\ell^w) \tag{31}$$

From the bargaining first order condition, we obtain

$$\begin{aligned} d(a_h, a_\ell) &= (1-\theta)[u_h(a_h + q^s + q^r) - u_h(a_h) + \beta V_h(a_h + q^s) - \beta \tilde{W}_h(a_h)] \\ &\quad -\theta[u_\ell(a_\ell - q^s - q^r) - u_\ell(a_\ell) + \beta V_\ell(a_\ell - q^s) - \beta \tilde{W}_\ell(a_\ell)] \end{aligned}$$

so that the transfer $d(\bar{a}_h, \bar{a}_\ell)$ is (using the fact that $q^s(\bar{a}_h, \bar{a}_\ell) = 0$),

$$\begin{aligned} d(\bar{a}_h, \bar{a}_\ell) &= (1-\theta)[u_h(\bar{a}_h + q^r) - u_h(\bar{a}_h) + \beta V_h(\bar{a}_h) - \beta \tilde{W}_h(\bar{a}_h)] \\ &\quad -\theta[u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell) - \beta \tilde{W}_\ell(\bar{a}_\ell)] \end{aligned}$$

Therefore, we obtain (using the relation between $\bar{a}_i$ and a_i^w as well as equation (31)):

$$\begin{aligned} u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h) &= u_h(\bar{a}_h) + \beta \tilde{W}_h(\bar{a}_h) \\ &\quad +\theta\left\{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_\ell) - u_h(\bar{a}_h)\right\} \end{aligned}$$

Also

$$\begin{aligned} u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell) &= u_\ell(\bar{a}_\ell) + \beta \tilde{W}_\ell(\bar{a}_\ell) \\ &\quad +(1-\theta)\left\{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_\ell) - u_h(\bar{a}_h)\right\} \end{aligned}$$

In a similar fashion, we obtain (using $q^s(\bar{a}_\ell, \bar{a}_h) = \bar{a}_h - \bar{a}_\ell$)

$$\begin{aligned} d(\bar{a}_\ell, \bar{a}_h) &= (1-\theta)[u_h(\bar{a}_h + q^r) - u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_h) - \beta \tilde{W}_h(\bar{a}_\ell)] \\ &\quad -\theta[u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_h) + \beta V_\ell(\bar{a}_\ell) - \beta \tilde{W}_\ell(\bar{a}_h)] \end{aligned}$$

Therefore,

$$\begin{aligned} u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell) &= u_\ell(\bar{a}_h) + \beta \tilde{W}_\ell(\bar{a}_h) \\ &\quad +(1-\theta)[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)] \end{aligned}$$

and similarly

$$\begin{aligned} u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h) &= u_h(\bar{a}_\ell) + \beta \tilde{W}_h(\bar{a}_\ell) \\ &\quad +\theta[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)] \end{aligned}$$

Using the above calculations, we obtain

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h) + \beta\tilde{W}_h(\bar{a}_h)] + (1-\pi)[u_\ell(\bar{a}_h) + \beta\tilde{W}_\ell(\bar{a}_h)] + \theta\pi S + (1-\theta)(1-\pi)\tilde{S} \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell) + \beta\tilde{W}_\ell(\bar{a}_\ell)] + (1-\pi)[u_h(\bar{a}_\ell) + \beta\tilde{W}_h(\bar{a}_\ell)] + \pi(1-\theta)S + (1-\pi)\theta\tilde{S} \end{aligned}$$

where

$$\begin{aligned} S &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_h) - u_\ell(\bar{a}_\ell) \\ \tilde{S} &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h) \end{aligned}$$

And taking the derivative, we have

$$V_h'(\bar{a}_h) = \pi[u_h'(\bar{a}_h) + \beta p] + (1-\pi)[u_\ell'(\bar{a}_h) + \beta p] + (1-\pi)(1-\theta)\frac{\partial \tilde{S}}{\partial \bar{a}_h} + \theta\pi\frac{\partial S}{\partial \bar{a}_h}$$

and using the first order condition for q^r we obtain

$$\begin{aligned} V_h'(\bar{a}_h) &= \beta p + (1-\pi)u_\ell'(\bar{a}_h) + \pi u_h'(\bar{a}_h) \\ &\quad + (1-\pi)(1-\theta)[u_h'(\bar{a}_h + q^r) - u_\ell'(\bar{a}_h)] \\ &\quad + \theta\pi[u_h'(\bar{a}_h + q^r) - u_h'(\bar{a}_h)] \end{aligned}$$

Also, using the first order condition for q^r we obtain

$$\begin{aligned} V_\ell'(\bar{a}_\ell) &= \beta p + \pi u_\ell'(\bar{a}_\ell) + (1-\pi)u_h'(\bar{a}_\ell) \\ &\quad + \pi(1-\theta)[u_\ell'(\bar{a}_\ell - q^r) - u_\ell'(\bar{a}_\ell)] + (1-\pi)\theta[u_\ell'(\bar{a}_\ell - q^r) - u_h'(\bar{a}_\ell)] \end{aligned}$$

The first condition for q^s imposes that $V_h'(\bar{a}_h) = V_\ell'(\bar{a}_\ell)$. Using the fact that $u_\ell'(\bar{a}_\ell - q^r) = u_h'(\bar{a}_h + q^r)$ and simplifying, we obtain

$$u_h'(\bar{a}_h + q^r) = \frac{\pi\theta u_\ell'(\bar{a}_\ell) + (1-\pi)(1-\theta)u_h'(\bar{a}_\ell) - (1-\pi)\theta u_\ell'(\bar{a}_h) - \pi(1-\theta)u_h'(\bar{a}_h)}{(1-2\pi)(1-2\theta)}$$

Therefore the equilibrium is given by

$$\begin{aligned}
u_h'(\bar{a}_h + q^r) &= u_\ell'(\bar{a}_\ell - q^r) \\
\bar{a}_h + \bar{a}_\ell &= 2A \\
u_h'(\bar{a}_h + q^r) &= \frac{\pi\theta u_\ell'(\bar{a}_\ell) + (1-\pi)(1-\theta)u_h'(\bar{a}_\ell) - (1-\pi)\theta u_\ell'(\bar{a}_h) - \pi(1-\theta)u_h'(\bar{a}_h)}{(1-2\pi)(1-2\theta)}
\end{aligned}$$

Notice that β does not impact the equilibrium allocation.

Also, suppose that $q^r = 0$ is an equilibrium. Then $u_h'(\bar{a}_h) = u_\ell'(\bar{a}_\ell)$ and

$$u_h'(\bar{a}_h)(1-2\theta) = (1-\theta)u_h'(\bar{a}_\ell) - \theta u_\ell'(\bar{a}_h)$$

or

$$\theta\left[u_\ell'(\bar{a}_h) - u_h'(\bar{a}_h)\right] = (1-\theta)\left[u_h'(\bar{a}_\ell) - u_h'(\bar{a}_h)\right]$$

However, since $\bar{a}_\ell \leq \bar{a}_h$ the RHS is positive, while the LHS is negative by assumption. Therefore, $q^r = 0$ cannot be an equilibrium.

Now we show that the amount of repo is actually lower under this arrangement. The equilibrium allocations under the benchmark and Walrasian outside-option, can be summarized by

$$\begin{aligned}
& (1-2\pi)(1-\beta)(1-2\theta)u_h'(c_h^*) \\
= {} & \pi(1-\beta)\left[\theta u_\ell'(c_\ell^* + \bar{q}^r) - (1-\theta)u_h'(c_h^* - \bar{q}^r)\right] \\
& -(1-\pi)(1-\beta)\left[\theta u_\ell'(c_h^* - \bar{q}^r) - (1-\theta)u_h'(c_\ell^* + \bar{q}^r)\right]
\end{aligned}$$

where $u_h'(c_h^*) = u_\ell'(c_\ell^*)$ with $c_h^* + c_\ell^* = 2A$ and q^{r*} and $\bar{q}^*$ are the repo amounts under the benchmark and Walrasian outside-option respectively. Define

$$LH(q^r) = \theta u_\ell'(c_\ell^* + q^r) - (1-\theta)u_h'(c_h^* - q^r)$$

and

$$RH(q^r) = \theta u_\ell'(c_h^* - q^r) - (1-\theta)u_h'(c_\ell^* + q^r).$$

Note that for $q^r \leq (c_h{}^* - c_\ell{}^*)/2$, we have

$$u_\ell'(c_h^* - q^r) < u_\ell'(c_\ell^* + q^r) < u_\ell'(c_\ell^*) = u_h'(c_h^*) < u_h'(c_h^* - q^r) < u_h'(c_\ell^* + q^r)$$

therefore we have $LH(q^r) > RH(q^r)$, which implies for all q^r,

$$[\pi(1-\beta) + (1-\pi)\beta]\, LH(q^r) - (1-\pi)RH(q^r) > \pi(1-\beta)LH(q^r) - (1-\pi)(1-\beta)RH(q^r)$$

Finally concavity of u_h and u_ℓ implies

$$\frac{d}{dq^r}LH(q^r) \quad < 0 < \quad \frac{d}{dq^r}RH(q^r),$$

hence $[\pi(1-\beta) + (1-\pi)\beta]\, LH(q^r) - (1-\pi)RH(q^r)$ and $\pi(1-\beta)LH(q^r) - (1-\pi)(1-\beta)RH(q^r)$ are both decreasing in q^r. This means we have

$$\begin{aligned}
& (1-2\pi)(1-\beta)(1-2\theta)u_h'(c_h^*) \\
= \; & [\pi(1-\beta) + (1-\pi)\beta]\, [\theta u_\ell'(c_\ell^* + q^{r*}) - (1-\theta)u_h'(c_h^* - q^{r*})] \\
& -(1-\pi)\, [\theta u_\ell'(c_h^* - q^{r*}) - (1-\theta)u_h'(c_\ell^* + q^{r*})] \\
> \; & \pi(1-\beta)\, [\theta u_\ell'(c_\ell^* + q^{r*}) - (1-\theta)u_h'(c_h^* - q^{r*})] \\
& -(1-\pi)(1-\beta)\, [\theta u_\ell'(c_h^* - q^{r*}) - (1-\theta)u_h'(c_\ell^* + q^{r*})]
\end{aligned}$$

therefore $q^{r*} > \bar{q}^r$. Note that the change in β does not affect $\bar{q}^r$, but as β approaches to zero, then $q^{r*} \to \bar{q}^r$.

Wedge

The solution for (19) implies

$$\begin{aligned}
u_h'(a_h' + q^r) &= u_\ell'(a_\ell' - q^r) \\
d^r &= (1-\theta_r)\, [u_h(a_h' + q^r) - u_h(a_h')] + \theta_r\, [u_\ell(a_\ell') - u_\ell(a_\ell' - q^r)]
\end{aligned}$$

hence

$$\begin{aligned}W_h^r(a_h') &= \theta_r\left[u_h(a_h'+q^r)+u_\ell(a_\ell'-q^r)\right]+\left[(1-\theta_r)u_h(a_h')-\theta_r u_\ell(a_\ell')\right]\\&\quad+\beta\left\{\pi\cdot W_h^s(a_h')+(1-\pi)\cdot W_\ell^s(a_h')\right\}.\\W_\ell^r(a_\ell') &= (1-\theta_r)\left[u_h(a_h'+q^r)+u_\ell(a_\ell'-q^r)\right]-\left[(1-\theta_r)u_h(a_h')-\theta_r u_\ell(a_\ell')\right]\\&\quad+\beta\left\{\pi\cdot W_\ell^s(a_\ell')+(1-\pi)\cdot W_h^s(a_\ell')\right\}.\end{aligned}$$

The solution for (18) implies

$$\begin{aligned}W_h^{r\prime}(a_h+q^s) &= \omega\cdot W_\ell^{r\prime}(a_\ell-q^s)\\d^s &= \frac{1}{\omega}\left\{(1-\theta_s)\left[W_h^r(a_h+q^s)-W_h^r(a_h)\right]-\omega\theta_s\left[W_\ell^r(a_\ell-q^s)-W_\ell^r(a_\ell)\right]\right\}\end{aligned}$$

hence

$$\begin{aligned}W_h^s(a_h) &= \theta_s\left[W_h^r(a_h+q^s)+\omega W_\ell^r(a_\ell-q^s)\right]+\left[(1-\theta_s)W_h^r(a_h)-\omega\theta_s W_\ell^r(a_\ell)\right]\\W_\ell^s(a_\ell) &= \frac{1}{\omega}(1-\theta_s)\left[W_h^r(a_h+q^s)+\omega W_\ell^r(a_\ell-q^s)\right]-\frac{1}{\omega}\left[(1-\theta_s)W_h^r(a_h)-\omega\theta_s W_\ell^r(a_\ell)\right]\end{aligned}$$

therefore

$$\begin{aligned}W_h^r(a_h') &= \theta_r\left[u_h(a_h'+q^r)+u_\ell(a_\ell'-q^r)\right]+\left[(1-\theta_r)u_h(a_h')-\theta_r u_\ell(a_\ell')\right]\\&\quad+\beta\pi\left\{\theta_s\left[W_h^r(a_h'+q^s)+\omega W_\ell^r(a_\ell'-q^s)\right]+\left[(1-\theta_s)W_h^r(a_h')-\omega\theta_s W_\ell^r(a_\ell')\right]\right\}\\&\quad+\beta(1-\pi)\left\{\frac{1}{\omega}(1-\theta_s)\left[W_h^r(a_h''+q^s)+\omega W_\ell^r(a_h'-q^s)\right]-\frac{1}{\omega}\left[(1-\theta_s)W_h^r(a_h'')-\omega\theta_s W_\ell^r(a_h')\right]\right\}\\W_\ell^r(a_\ell') &= (1-\theta_r)\left[u_h(a_h'+q^r)+u_\ell(a_\ell'-q^r)\right]-\left[(1-\theta_r)u_h(a_h')-\theta_r u_\ell(a_\ell')\right]\\&\quad+\beta\pi\left\{\frac{1}{\omega}(1-\theta_s)\left[W_h^r(a_h'+q^s)+\omega W_\ell^r(a_\ell'-q^s)\right]-\frac{1}{\omega}\left[(1-\theta_s)W_h^r(a_h')-\omega\theta_s W_\ell^r(a_\ell')\right]\right\}\\&\quad+\beta(1-\pi)\left\{\theta_s\left[W_h^r(a_\ell'+q^s)+\omega W_\ell^r(a_\ell''-q^s)\right]+\left[(1-\theta_s)W_h^r(a_\ell')-\omega\theta_s W_\ell^r(a_\ell'')\right]\right\}.\end{aligned}$$

where in the equilibrium sale market, a type h agent with asset holding a_h' is matched with a type ℓ agent with asset holding a_ℓ', a type h agent with asset holding a_ℓ' is matched with a type ℓ agent with asset holding a_ℓ'', and type ℓ agent with asset holding a_h' is matched with a type h agent with asset holding a_h''.

Now assuming that by a marginal change in the asset holding the counter-party's asset holding remains constant, and also applying of the Envelop theorem for the effects on q^r and

q^s we have

$$\begin{aligned}
W_h^{r\prime}(a'_h) &= \theta_r u'_h(a'_h + q^r) + (1-\theta_r) u'_h(a'_h) \\
&\quad + \beta\pi\left\{\theta_s W_h^{r\prime}(a'_h + q^s) + (1-\theta_s) W_h^{r\prime}(a'_h)\right\} \\
&\quad + \beta(1-\pi)\left\{(1-\theta_s) W_\ell^{r\prime}(a'_h - q^s) + \theta_s W_\ell^{r\prime}(a'_h)\right\} \\
W_\ell^{r\prime}(a'_\ell) &= (1-\theta_r) u'_\ell(a'_\ell - q^r) + \theta_r u'_\ell(a'_\ell) \\
&\quad + \beta\pi\left\{(1-\theta_s) W_\ell^{r\prime}(a'_\ell - q^s) + \theta_s W_\ell^{r\prime}(a'_\ell)\right\} \\
&\quad + \beta(1-\pi)\left\{\theta_s W_h^{r\prime}(a'_\ell + q^s) + (1-\theta_s) W_h^{r\prime}(a'_\ell)\right\}.
\end{aligned}$$

In the equilibrium we have type h agents would adjust their asset holdings to a_h^* and type ℓ agents would adjust their asset holdings to a_ℓ^* after realizing their type, where $a_h^* + a_\ell^* = 2A$ and $W_h^{r\prime}(a_h^*) = \omega \cdot W_\ell^{r\prime}(a_\ell^*)$, which implies $q^s(a_h^*, a_\ell^*) = 0$ and $q^s(a_\ell^*, a_h^*) = a_h^* - a_\ell^*$. Moreover, we have $q^r(a_h^*, a_\ell^*) = c_h^* - a_h^*$, where $u'_h(c_h^*) = u'_\ell(2A - c_h^*) = u'_\ell(c_\ell^*)$. Therefore, we have

$$\begin{aligned}
W_h^{r\prime}(a_h^*) &= \theta_r u'_h(c_h^*) + (1-\theta_r) u'_h(a_h^*) \\
&\quad + \beta\pi\left\{\theta_s W_h^{r\prime}(a_h^*) + (1-\theta_s) W_h^{r\prime}(a_h^*)\right\} \\
&\quad + \beta(1-\pi)\left\{(1-\theta_s) W_\ell^{r\prime}(a_\ell^*) + \theta_s W_\ell^{r\prime}(a_h^*)\right\} \\
W_\ell^{r\prime}(a_\ell^*) &= (1-\theta_r) u'_\ell(c_\ell^*) + \theta_r u'_\ell(a_\ell^*) \\
&\quad + \beta\pi\left\{(1-\theta_s) W_\ell^{r\prime}(a_\ell^*) + \theta_s W_\ell^{r\prime}(a_\ell^*)\right\} \\
&\quad + \beta(1-\pi)\left\{\theta_s W_h^{r\prime}(a_h^*) + (1-\theta_s) W_h^{r\prime}(a_\ell^*)\right\} \\
W_h^{r\prime}(a_\ell^*) &= \theta_r u'_h(a_\ell^* + q^r(a_\ell^*, a_\ell^*)) + (1-\theta_r) u'_h(a_\ell^*) \\
&\quad + \beta\pi\left\{\theta_s W_h^{r\prime}(a_h^*) + (1-\theta_s) W_h^{r\prime}(a_\ell^*)\right\} \\
&\quad + \beta(1-\pi)\left\{(1-\theta_s) W_\ell^{r\prime}(a_\ell^*) + \theta_s W_\ell^{r\prime}(a_\ell^*)\right\} \\
W_\ell^{r\prime}(a_h^*) &= (1-\theta_r) u'_\ell(a_h^* - q^r(a_h^*, a_h^*)) + \theta_r u'_\ell(a_h^*) \\
&\quad + \beta\pi\left\{(1-\theta_s) W_\ell^{r\prime}(a_\ell^*) + \theta_s W_\ell^{r\prime}(a_h^*)\right\} \\
&\quad + \beta(1-\pi)\left\{\theta_s W_h^{r\prime}(a_h^*) + (1-\theta_s) W_h^{r\prime}(a_h^*)\right\}
\end{aligned}$$

where $q^r(a_\ell^*, a_\ell^*)$ and $q^r(a_h^*, a_h^*)$ are set such that $u'_h(a_\ell^* + q^r(a_\ell^*, a_\ell^*)) = u'_\ell(a_\ell^* - q^r(a_\ell^*, a_\ell^*))$ and $u'_h(a_h^* + q^r(a_h^*, a_h^*)) = u'_\ell(a_h^* - q^r(a_h^*, a_h^*))$. Note that in the equilibrium type i agents would enter the repo market with asset holding a_i^* for $i \in \{h, \ell\}$, and therefore when a type i agent shows up in the repo market with asset holding a_{-i}^*, she will be still matched with a $-i$ agent with asset holding a_{-i}^*.

This implies

$$
\begin{aligned}
(1-\beta\pi)\cdot W_h^{r\prime}(a_h^*) &= \theta_r u_h'(c_h^*) + (1-\theta_r)u_h'(a_h^*) \\
&\quad + \beta(1-\pi)\left\{(1-\theta_s)W_\ell^{r\prime}(a_\ell^*) + \theta_s W_\ell^{r\prime}(a_h^*)\right\} \\
(1-\beta\pi)\cdot W_\ell^{r\prime}(a_\ell^*) &= (1-\theta_r)u_\ell'(c_\ell^*) + \theta_r u_\ell'(a_\ell^*) \\
&\quad + \beta(1-\pi)\left\{\theta_s W_h^{r\prime}(a_h^*) + (1-\theta_s)W_h^{r\prime}(a_\ell^*)\right\} \\
(1-\beta\pi(1-\theta_s))\cdot W_h^{r\prime}(a_\ell^*) &= \theta_r u_h'(a_\ell^* + q^r(a_\ell^*, a_\ell^*)) + (1-\theta_r)u_h'(a_\ell^*) \\
&\quad + \beta\pi\theta_s W_h^{r\prime}(a_h^*) + \beta(1-\pi)W_\ell^{r\prime}(a_\ell^*) \\
(1-\beta\pi\theta_s)\cdot W_\ell^{r\prime}(a_h^*) &= (1-\theta_r)u_\ell'(a_h^* - q^r(a_h^*, a_h^*)) + \theta_r u_\ell'(a_h^*) \\
&\quad + \beta\pi(1-\theta_s)W_\ell^{r\prime}(a_\ell^*) + \beta(1-\pi)W_h^{r\prime}(a_h^*)
\end{aligned}
$$

hence

$$
\begin{aligned}
(1-\beta\pi)\cdot W_h^{r\prime}(a_h^*) &= \left\{\theta_r u_h'(c_h^*) + (1-\theta_r)u_h'(a_h^*)\right\} \\
&\quad + \beta(1-\pi)(1-\theta_s)W_\ell^{r\prime}(a_\ell^*) \\
&\quad + \frac{\beta(1-\pi)\theta_s}{(1-\beta\pi\theta_s)}\left\{(1-\theta_r)u_\ell'(a_h^* - q^r(a_h^*, a_h^*)) + \theta_r u_\ell'(a_h^*)\right\} \\
&\quad + \frac{\beta(1-\pi)\theta_s}{(1-\beta\pi\theta_s)}\left\{\beta\pi(1-\theta_s)W_\ell^{r\prime}(a_\ell^*) + \beta(1-\pi)W_h^{r\prime}(a_h^*)\right\} \\
(1-\beta\pi)\cdot W_\ell^{r\prime}(a_\ell^*) &= \left\{(1-\theta_r)u_\ell'(c_\ell^*) + \theta_r u_\ell'(a_\ell^*)\right\} \\
&\quad + \beta(1-\pi)\theta_s W_h^{r\prime}(a_h^*) \\
&\quad + \frac{\beta(1-\pi)(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}\left\{\theta_r u_h'(a_\ell^* + q^r(a_\ell^*, a_\ell^*)) + (1-\theta_r)u_h'(a_\ell^*)\right\} \\
&\quad + \frac{\beta(1-\pi)(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}\left\{\beta\pi\theta_s W_h^{r\prime}(a_h^*) + \beta(1-\pi)W_\ell^{r\prime}(a_\ell^*)\right\}
\end{aligned}
$$

which means

$$\begin{aligned}
&\left\{(1-\beta\pi)-\frac{\beta(1-\pi)\beta(1-\pi)\theta_s}{(1-\beta\pi\theta_s)}\right\}\cdot W_h^{r\prime}(a_h^*) \\
&-\beta(1-\pi)(1-\theta_s)\left\{1+\frac{\beta\pi\theta_s}{(1-\beta\pi\theta_s)}\right\}W_\ell^{r\prime}(a_\ell^*) = \left\{\theta_r u_h'(c_h^*)+(1-\theta_r)u_h'(a_h^*)\right\} \\
&\qquad +\frac{\beta(1-\pi)\theta_s}{(1-\beta\pi\theta_s)}(1-\theta_r)u_\ell'(a_h^*-q^r(a_h^*,a_h^*)) \\
&\qquad +\frac{\beta(1-\pi)\theta_s}{(1-\beta\pi\theta_s)}\theta_r u_\ell'(a_h^*)
\end{aligned}$$

$$\begin{aligned}
&\left\{(1-\beta\pi)-\frac{\beta(1-\pi)\beta(1-\pi)(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}\right\}\cdot W_\ell^{r\prime}(a_\ell^*) \\
&-\beta(1-\pi)\theta_s\left\{1+\frac{\beta\pi(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}\right\}W_h^{r\prime}(a_h^*) = \left\{(1-\theta_r)u_\ell'(c_\ell^*)+\theta_r u_\ell'(a_\ell^*)\right\} \\
&\qquad +\frac{\beta(1-\pi)(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}\theta_r u_h'(a_\ell^*+q^r(a_\ell^*,a_\ell^*)) \\
&\qquad +\frac{\beta(1-\pi)(1-\theta_s)}{(1-\beta\pi(1-\theta_s))}(1-\theta_r)u_h'(a_\ell^*)
\end{aligned}$$

In turn

$$\begin{aligned}
&(1-\beta\pi)(1-\beta\pi\theta_s)W_h^{r\prime}(a_h^*) \\
&-\beta(1-\pi)\beta(1-\pi)\theta_s W_h^{r\prime}(a_h^*) \\
&-\beta(1-\pi)(1-\theta_s)W_\ell^{r\prime}(a_\ell^*) = (1-\beta\pi\theta_s)\left\{\theta_r u_h'(c_h^*)+(1-\theta_r)u_h'(a_h^*)\right\} \\
&\qquad +\beta(1-\pi)\theta_s\left\{(1-\theta_r)u_\ell'(a_h^*-q^r(a_h^*,a_h^*))+\theta_r u_\ell'(a_h^*)\right\}
\end{aligned}$$

and

$$\begin{aligned}
&(1-\beta\pi)(1-\beta\pi(1-\theta_s))W_\ell^{r\prime}(a_\ell^*) \\
&-\beta(1-\pi)\beta(1-\pi)(1-\theta_s)W_\ell^{r\prime}(a_\ell^*) \\
&-\beta(1-\pi)\theta_s W_h^{r\prime}(a_h^*) = (1-\beta\pi(1-\theta_s))\left\{(1-\theta_r)u_\ell'(c_\ell^*)+\theta_r u_\ell'(a_\ell^*)\right\} \\
&\qquad +\beta(1-\pi)(1-\theta_s)\left\{\theta_r u_h'(a_\ell^*+q^r(a_\ell^*,a_\ell^*))+(1-\theta_r)u_h'(a_\ell^*)\right\}
\end{aligned}$$

In equilibrium we have $W_h^{r\prime}(a_h^*)=\omega\cdot W_\ell^{r\prime}(a_\ell^*)$, a_h^* and a_ℓ^* are given by $a_h^*=2A-a_\ell^*$ and

$$\frac{\begin{array}{c}(1-\beta\pi(1-\theta_s))\left\{(1-\theta_r)u_\ell'\left(c_\ell^*\right)+\theta_r u_\ell'\left(a_\ell^*\right)\right\}\\ +\beta(1-\pi)(1-\theta_s)\left\{\theta_r u_h'\left(a_\ell^*+q^r(a_\ell^*,a_\ell^*)\right)+(1-\theta_r)u_h'\left(a_\ell^*\right)\right\}\end{array}}{\left\{(1-\beta\pi)(1-\beta\pi(1-\theta_s))-\beta(1-\pi)\beta(1-\pi)(1-\theta_s)\right\}-\beta(1-\pi)\theta_s\cdot\omega}$$

$$= \frac{\begin{array}{c}(1-\beta\pi\theta_s)\left\{\theta_r u_h'\left(c_h^*\right)+(1-\theta_r)u_h'\left(a_h^*\right)\right\}\\ +\beta(1-\pi)\theta_s\left\{(1-\theta_r)u_\ell'\left(a_h^*-q^r(a_h^*,a_h^*)\right)+\theta_r u_\ell'\left(a_h^*\right)\right\}\end{array}}{\left\{(1-\beta\pi)(1-\beta\pi\theta_s)-\beta(1-\pi)\beta(1-\pi)\theta_s\right\}\cdot\omega-\beta(1-\pi)(1-\theta_s)} \tag{32}$$

Now as ω increases, the denominator of the left side of equation (32) decreases and the denominator of the right side increases. The equality is maintained by the increase in the numerator of the left side and decrease in the numerator of the right side. Hence, by the concavity of $u(\cdot)$, an increase in the wedge ω decreases a_h^* and increases a_ℓ^*.

Proof of Proposition 10

If repo is not available and agents can only buy and sell their assets, then each agent will start the next period with the amount of asset that he consumes in the current period. Hence the value of asset holding a for a type B agent with shock h is given by

$$V_{B,h}(a) \quad = \quad u_{B,h}(a+q^s) - d + \beta \cdot V_B(a+q^s)$$

where $V_B(a) = \pi \cdot V_{B,h}(a) + (1-\pi) \cdot V_{B,\ell}(a)$ and the terms of trade, q^s and d, are set as the solution for the following bargaining problem.

$$\max_{q^s,d} \quad \left[u_{B,h}(a_{B,h}+q^s) - d + \beta \cdot V_B(a_{B,h}+q^s) - u_{B,h}(a_{B,h}) - \beta \cdot V_B(a_{B,h})\right]^{\theta} \tag{33}$$
$$\times \left[u_{L,\ell}(a_{L,\ell}-q^s) + d + \beta \cdot V_L(a_{L,\ell}-q^s) - u_{L,\ell}(a_{L,\ell}) - \beta \cdot V_L(a_{L,\ell})\right]^{1-\theta}$$

$$s.t. \quad q^s \in \left[-a_{B,h}, a_{L,\ell}\right].$$

The value functions $V_{B,\ell}(\cdot)$, $V_{L,h}(\cdot)$, $V_{L,\ell}(\cdot)$, and $V_L(\cdot)$ are defined similarly.

The solution for the bargaining problem (33) and the similar problem for the bargaining between (B,ℓ) and (L,h) agents imply that

$$\begin{aligned} u'_{B,h}\left(a_{B,h}+q^s_{h,\ell}\right)+\beta\cdot V'_B\left(a_{B,h}+q^s_{h,\ell}\right) &= u'_{L,\ell}\left(a_{L,\ell}-q^s_{h,\ell}\right)+\beta\cdot V'_L\left(a_{L,\ell}-q^s_{h,\ell}\right) \qquad (34)\\ u'_{B,\ell}\left(a_{B,\ell}+q^s_{\ell,h}\right)+\beta\cdot V'_B\left(a_{B,\ell}+q^s_{\ell,h}\right) &= u'_{L,h}\left(a_{L,h}-q^s_{\ell,h}\right)+\beta\cdot V'_L\left(a_{L,h}-q^s_{\ell,h}\right). \end{aligned}$$

If the equilibrium allocation is efficient then the distribution of type B agents' asset holdings should have two mass points on $c^*_{B,h}$ and $c^*_{B,\ell}$ with probabilities π and $1-\pi$, and the distribution of type L agents' asset holdings should have two mass points on $c^*_{L,h}$ and $c^*_{L,\ell}$ with probabilities $1-\pi$ and π, where $c^*_{B,h}$, $c^*_{B,\ell}$, $c^*_{L,h}$, and $c^*_{L,\ell}$ are defined by (20). In this efficient equilibrium, (B,h) agents with asset holding of $c^*_{B,h}$ do not trade. But a (B,h) agent with asset holding of $c^*_{B,\ell}$ search and match with a (L,ℓ) agent with asset holding $c^*_{L,h}$ and they adjust their asset holdings to $c^*_{B,h}$ and $c^*_{L,\ell}$. Hence in the equilibrium we have

$$V_{B,h}(c_{B,\ell}) = u(c_{B,\ell}+q)+\beta V(c_{B,\ell}+q)-d(c_{B,\ell},c_{L,h})$$

where

$$\begin{aligned} d &= (1-\theta)S_{B,h}(c_{B,\ell},c_{L,h})-\theta S_{L,\ell}(c_{B,\ell},c_{L,h})\\ &= (1-\theta)[u_{B,h}(c_{B,\ell}+q)+\beta V_B(c_{B,\ell}+q)-u_{B,h}(c_{B,\ell})-\beta V_B(c_{B,\ell})]\\ &\quad -\theta[u_{L,\ell}(c_{L,h}-q)+\beta V_L(c_{L,h}-q)-u_{L,\ell}(c_{L,h})-\beta V_L(c_{L,h})] \end{aligned}$$

Therefore, we obtain

$$\begin{aligned} V'_{B,h}(c^*_{B,h}) &= u'_{B,h}(c^*_{B,h})+\beta\cdot V'_B(c^*_{B,h})\\ V'_{B,h}(c^*_{B,\ell}) &= \theta\left[u'_{B,h}(c^*_{B,h})+\beta\cdot V'_B(c^*_{B,h})\right]+(1-\theta)\left[u'_{B,h}(c^*_{B,\ell})+\beta\cdot V'_B(c^*_{B,\ell})\right]\\ V'_{B,\ell}(c^*_{B,\ell}) &= u'_{B,\ell}(c^*_{B,\ell})+\beta\cdot V'_B(c^*_{B,\ell})\\ V'_{B,\ell}(c^*_{B,h}) &= \theta\left[u'_{B,\ell}(c^*_{B,\ell})+\beta\cdot V'_B(c^*_{B,\ell})\right]+(1-\theta)\left[u'_{B,\ell}(c^*_{B,h})+\beta\cdot V'_B(c^*_{B,h})\right] \end{aligned}$$

which implies

$$\begin{aligned} V'_B(c^*_{B,h}) &= \pi\cdot u'_{B,h}(c^*_{B,h})+(1-\pi)\theta\cdot u'_{B,\ell}(c^*_{B,\ell})+(1-\pi)(1-\theta)\cdot u'_{B,\ell}(c^*_{B,h})\\ &\quad +\beta\left(\pi+(1-\pi)(1-\theta)\right)\cdot V'_B(c^*_{B,h})+\beta(1-\pi)\theta\cdot V'_B(c^*_{B,\ell}). \qquad (35) \end{aligned}$$

Similarly for type L, we have

$$\begin{aligned} V_L'(c_{L,\ell}^*) &= \pi \cdot u_{L,\ell}'(c_{L,\ell}^*) + (1-\pi)(1-\theta) \cdot u_{L,h}'(c_{L,h}^*) + (1-\pi)\theta \cdot u_{L,h}'(c_{L,\ell}^*) \\ &\quad + \beta\left(\pi + (1-\pi)\theta\right) \cdot V_L'(c_{L,\ell}^*) + \beta(1-\pi)(1-\theta) \cdot V_L'(c_{L,h}^*). \end{aligned} \tag{36}$$

Using (20) and the bargaining solutions (34) in equilibrium we have

$$\begin{aligned} V_B'(c_{B,h}^*) &= V_L'(c_{L,\ell}^*) \\ V_B'(c_{B,\ell}^*) &= V_L'(c_{L,h}^*). \end{aligned}$$

Therefore, subtracting (35) from (36) we should have

$$\begin{aligned} & \theta\left[u_{L,h}'(c_{L,\ell}^*) + u_{B,\ell}'(c_{B,h}^*)\right] \\ = \;& (2\theta-1)\left[u_{B,\ell}'(c_{B,\ell}^*) + u_{B,\ell}'(c_{B,h}^*) + \beta \cdot \left\{V_B'(c_{B,\ell}^*) - V_B'(c_{B,h}^*)\right\}\right]. \end{aligned}$$

Note that for $\theta \leq \frac{1}{2}$ the left hand of the above equality is positive, while the right hand of it is not. Hence, it cannot hold. Note that this is true for any $\pi \in [0,1]$.

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