Money Talks*

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Abstract

We study credible information transmission by a benevolent short-lived central bank. When externalities create a wedge between private and social welfare, the central bank has an incentive to mis-report its information. In contrast, information transmission through monetary policy creates a distortion, thus, lending credibility.

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1 Introduction

What is the role of monetary policy as a tool for information revelation in the presence of potential coordination problems faced by investors? Why do central banks typically follow policies that lead to positive average levels of inflation? These questions have a long history in monetary economics. However, decisive answers still elude us. We investigate monetary policy as a tool for credible information transmission by the central bank.

Our model contains the following three ingredients: (1) money plays a role in facilitating trade, (2) there is aggregate risk about fundamentals, and (3) investment decisions are subject to a coordination problem, implying that individually optimal decisions might not maximize aggregate welfare. Investors have expectations about economic fundamentals that affect investment returns. The CB has private information about the true state of these fundamentals. We demonstrate that this information cannot always be credibly transmitted to the private sector. In other words, a simple announcement might not be enough, as the CB may prefer to communicate false information if this would lead to more socially desirable behavior by investors. We then show that credible information revelation is possible through monetary policy. In order to gain credibility, however, it is *necessary* that such a policy create a distortion by affecting average inflation.[1]

2 The Environment

We use the set-up in Berentsen and Monnet (2008), who build on Lagos and Wright (2005). Time $t = 0, 1, ...,$ is infinite. There is a continuum of infinitely-lived agents and a benevolent central bank (CB) that has the ability to make public announcements, print money, and make loans. The CB serves for one term and is replaced by a new CB at the end of each

[1] Needless to say, monetary policy in the actual economy serves several purposes. In order to concentrate on monetary policy's role as a credible communication device, we will abstract from effects related to liquidity provision. Informational effects of policy interventions in the global games environment is analyzed in Angeletos, Hellwig and Pavan (2006). Other related papers include: Barro and Gordon (1983), Backus and Driffill (1985), Blinder, Ehrmann, Fratzscher, De Haan, and Jansen (2008), Morris and Shin (2002), and Phelps (1983).

period.[2]

Each period contains three stages: 0, 1, and 2. There are three goods: an *investment good*, k, a *stage-1 good*, q, and a *stage-2 good*, z. Investment takes place in stage 0 of each period. The market for good q opens in stage 1. In stage 2, investment pays off in units of good z. Future periods are discounted at rate $\beta \in (0, 1)$.

In stage 0, half of the agents are randomly chosen to be *investors*. The remaining $\frac{1}{2}$ become consumers. Each investor i chooses how much of an investment good, k, to produce. The utility cost to investor i from producing $k_i > 0$ units of the investment good is $c(k_i) = \frac{1}{2}(k_i - \gamma K)^2$, where $K = \int k_i di$ is the aggregate production of the investment good and $\gamma \in (-\infty, 1)$. Thus, investors' costs are subject to an externality for any $\gamma \neq 0$. The cost is zero if $k_i = 0$. Investments mature in stage 2. The return on this investment is uncertain and is given by θ^2 units of good z per unit of k, where θ is a random variable with an improper uniform prior on $(-\infty, +\infty)$ and is *iid* across periods. Nature draws θ at the start of stage 0.[3]

The random variable θ introduces aggregate risk about the profitability of investment. When the true state is θ, the CB receives a signal $y = \theta + \eta$, while investors receives a signal $x = \theta + \varepsilon$. Signals are received in stage 0 of each period, prior to the investment decisions, and are *iid* across time.[4] The noise terms η and ε are normally distributed with mean zero and respective precisions given by $\alpha = 1/\sigma_\eta^2$ and $\delta = 1/\sigma_\varepsilon^2$. We do not take a stand on whether the CB or the private investors have more accurate information.

In stage 1, investors produce the stage-1 good, q, at a cost $c(q) = -q$. The consumers consume q, deriving utility $u(q)$, where u is increasing, concave, and satisfies the usual Inada conditions. In particular, there is a unique q^* such that $u'(q^*) = 1$. Good q is sold in a competitive market at price p. Traders are anonymous in this market, so money is used in the exchange of q. In stage 1 of each period, agents have access to a lending facility operated

[2]This allows us to abstract from reputation effects, which have been the focus of study in other papers. Our analysis focuses on achieving credible truthful communication in the absence of such reputation effects.

[3]We consider the return on investment to be θ^2 instead of θ simply in order to guarantee that investment is always positive. In our formulation with $\gamma \in (0, 1)$, the externality is essentially the same as in Angeletos and Pavan (2004), since the individual return on investment also increases in the aggregate level of investment.

[4]Assuming that investors' signals are perfectly correlated is for simplicity only. What is important is that the CB's announcement/interest rate choice takes place prior to the investment decisions.

by the CB, where they can borrow money at an interest rate $r \geq 0$. Loans are settled in stage 2.[5] The interest rate is announced by the CB at the start of stage 0, before investors make their investment decisions.

Finally, stage-0 investment delivers a return of $\theta^2 k_i$ units of the non-storable stage-2 good, z, which is traded in a competitive market in stage 2. The utility (disutility) of consumption (production) from z is denoted by $(-)z$. Stage 2 can be thought of as a settlement stage. Those consumers who borrowed money in stage 1 must produce in order to pay off their loans. The investors, who produce in stage 1, will end up consuming in stage 2.[6]

First, we consider the full-information benchmark. Aggregate period-t welfare, $\mathcal{W}$, is given by

$$\mathcal{W}(q, k_i, \theta) = \frac{1}{2}\left[u\left(q\right) - q + \theta^2 K - \int \frac{1}{2}\left(k_i - \gamma K\right)^2 di\right]. \tag{1}$$

Imposing that $k_i = K$, the first-order conditions for the efficient allocation (q^*, K^*) yield

$$u'\left(q^*\right) = 1, \text{ and} \tag{2}$$

$$K^* = \frac{\theta^2}{\left(1-\gamma\right)^2}. \tag{3}$$

To decentralize efficient allocations, assume that money is provided exclusively by the CB. We let M denote the per capita supply of money and ϕ be the real price of money in terms of z. The growth rate of money is given by ρ. Monetary injections are implemented through a transfer, T, in the settlement stage. The net stock of money grows according to $M_+ = M + T$, where subscript $+$ denotes next period values and T is such that $M_+ = \rho M$. We consider a stationary equilibrium where $\phi M = \phi_+ M_+$, so that $\rho = \phi/\phi_+$. We use $W(k, m, l; \theta)$ to denote the discounted lifetime utility of an agent entering stage 2 holding k units of the investment good, m units of money, and l units of loans from the CB, given that the realized productivity shock is θ. The function $V(m)$ denotes the expected discounted lifetime utility from entering stage 0 with money holdings m. Since $k_i = k = K$ for all

[5]For simplicity, we assume that the CB makes a lump-sum transfer in the settlement market in order to redistribute any profits made by its lending facility.

[6]Due to linearity, agents will exit stage 2 with equal money holdings. This dramatically improves tractability.

agents, the equilibrium aggregate production of the investment good under full information is given by $K = \theta^2 / (1-\gamma)$. Clearly, $K < K^*$ whenever $0 < \gamma < 1$, while $K = K^*$ whenever $\gamma = 0$, and $K > K^*$ whenever $\gamma < 0$. Hence, whenever $\gamma \neq 0$, the equilibrium level of production is different from the level that maximizes aggregate welfare, K^*.

The first-order conditions give

$$u'(q) = 1 + r. \tag{4}$$

The rate of inflation, ρ, is given by: $\frac{\rho}{\beta} = 1 + \frac{E[r]}{2}$. When the true value of θ is publicly observable, a benevolent CB must follow the Friedman rule; i.e., it sets $r = 0$ for all states and $u'(q^*) = 1$. Still, the CB does not achieve the efficient allocation as (3) is not satisfied.

3 Information Transmission

We turn to the case where θ is unknown. Both the private sector and the CB receive informative signals regarding the true value of θ. The private sector receives signal x with precision δ. The CB receives signal y with precision α. Suppose the CB reveals that its signal is y_a and that the precision of the signal is α_a. The first-order conditions for investors become

$$\begin{aligned} p\phi &= 1, \text{ and} && (5) \\ k(x, y_a, \alpha_a) &= E\left(\theta^2 + \gamma K \mid x, y_a, \alpha_a\right). && (6) \end{aligned}$$

Since all investors have the same information, we concentrate on symmetric outcomes where they all produce the same amount, $k(x, y_a, \alpha_a) = K(x, y_a, \alpha_a)$. Hence,

$$k(x, y_a, \alpha_a) = \frac{E(\theta^2 | x, y_a, \alpha_a)}{1-\gamma} = \frac{1}{1-\gamma}\left[\left(\frac{\alpha_a y_a + \delta x}{\alpha_a + \delta}\right)^2 + \frac{1}{\alpha_a + \delta}\right]. \tag{7}$$

Taking as given the behavior by the private sector, the CB maximizes expected period-t

welfare given its signal, y, and precision, α:

$$E\mathcal{W}(q,k_i)=\frac{1}{2}E\left[u\left(q\right)-q+\theta^2K-\int\frac{\left(k_i-\gamma K\right)^2}{2}di\mid\alpha,y\right]. \tag{8}$$

We will consider two possibilities regarding information revelation. First, the CB could make a direct "announcement." Alternatively, the CB could indirectly transmit its information through the interest rate, r. As the objective of the benevolent CB and that of an individual investors are not always aligned, the CB might have an incentive to misrepresent its information if that would lead to improved welfare. In contrast, transmission through the interest rate is not "cheap-talk," as it reduces welfare through a distortion.

3.1 Public Announcements

We assume that the precision of the CB's signal, α, is *iid* across periods and that it can take on two values: $\{\alpha_L,\alpha_H\}$, where $\alpha_L=\alpha_H-\varepsilon$, $\varepsilon>0$. The probability that $\alpha=\alpha_L$ is denoted by π, and the probability that $\alpha=\alpha_H$ by $1-\pi$. The realization of the CB's signal precision is observed only by the CB. To further simplify the analysis, we assume that y is publicly observable. In this case, the CB's announcement concerns only the value of its confidence. We denote such an announcement when the realized precision is α by $\alpha_a\left(\alpha\right)$.[7]

A *symmetric equilibrium* consists of actions, (α_a,k), and beliefs, (μ_B,μ_i), for the CB and the private sector, such that (a) The CB chooses α_a and has beliefs μ_B such that $E^{\mu,(\alpha_a,k)}[\mathcal{W}(q,k_i)|h]\geq E^{\mu,(\alpha_a',k)}[\mathcal{W}(q,k_i)|h]$, for any α_a', where μ_B is consistent with k; and (b) every i chooses k_i and has beliefs μ_i such that $E^{\mu,(k_i,A_{-i})}[u_i|h]\geq E^{\mu,(k_i',A_{-i})}[u_i|h]$, for any k_i', where $A_{-i}=(\alpha_a,k_{-i})$ and μ_i is consistent with (k_i',A_{-i}).[8]

The following asserts that there are cases such that if the investors believe the CB's announcement, expected social welfare is increased if the CB *misrepresents* its confidence.

Proposition 1 *Whenever $\gamma\neq0$, there is no equilibrium where the CB announces its preci-*

[7]We concentrate on the revelation of the CB's precision rather than y itself for technical simplicity. The fact that precision takes two possible values, as opposed to a continuum, greatly simplifies the analysis.

[8]See Kreps and Wilson (1982) for a formal definition of sequential equilibrium. In what follows we adopt their notation, using h to denote the respective information sets.

sion truthfully and where the investors use the CB's announcement. More precisely, suppose that μ_B and μ_i are consistent with $E[K \mid \alpha_a, \alpha, y]$. Then: (a) if $\gamma \in (0,1)$, there is an $\varepsilon > 0$ such that the CB prefers to under-report its precision; i.e. $E^{\mu,(\alpha_H-\varepsilon,k)}[\mathcal{W}(q,k_i)|h] > E^{\mu,(\alpha_H,k)}[\mathcal{W}(q,k_i)|h]$; (b) if $\gamma < 0$, there is an $\varepsilon > 0$ such that the CB prefers to over-report its precision; i.e. $E^{\mu,(\alpha_L+\varepsilon,k)}[\mathcal{W}(q,k_i)|h] > E^{\mu,(\alpha_L,k)}[\mathcal{W}(q,k_i)|h]$; and (c) if $\gamma = 0$, the CB reports the truth, i.e. $E^{\mu,(\alpha,k)}[\mathcal{W}(q,k_i)|h] > E^{\mu,(\alpha'_a,k)}[\mathcal{W}(q,k_i)|h]$, for all α'_a.

Given its announcement, α_a, the CB expects the following aggregate level of investment:

$$E[K \mid \alpha_a, \alpha, y] = \frac{y^2}{1-\gamma} + \frac{\delta^2 + 2\alpha\delta + \alpha\alpha_a}{\alpha(\alpha_a+\delta)^2(1-\gamma)}$$

This is decreasing with α_a for all y. Intuitively, when $\gamma \in (0,1)$, announcing a lower precision increases the expected return on the investment good. This brings the aggregate investment level, K, closer to the socially optimum. Similarly, when $\gamma < 0$, over-reporting depresses both the individual and aggregate investment decisions, bringing K closer to the optimum. The bigger the wedge between social and private welfare, the higher is the CB's incentive to lie ($\partial^2 E\mathcal{W}/(\partial\alpha_a\partial\gamma) < 0$). On the other hand, the CB's incentive to lie decreases as the precision of the private signal increases ($|\partial^2 E\mathcal{W}(q,k_i)/\partial\alpha_a\partial\delta\ |_{\alpha=\alpha_a}| > 0$ for $\delta > \alpha$).

3.2 Credible Monetary Policy

The CB chooses an interest rate rule, $r(\alpha)$, to reveal the precision of its signal. By "inverting" r, investors can infer the value of α that the CB wishes to communicate. The CB maximizes (8) subject to 5 and 6. In order for monetary policy to *credibly* transmit information, a set of incentive constraints must hold. Consider the case where the CB with $\alpha = \alpha_H$ prefers to mimic a CB with $\alpha = \alpha_L$, $\alpha_L < \alpha_H$ (this would be the case under the conditions in Proposition 1 for $\gamma > 0$; the $\gamma < 0$ case is analogous). Clearly, incentive compatibility does not bind if $\alpha = \alpha_L$. However, when $\alpha = \alpha_H$, the CB has an incentive to misrepresent its confidence in its information. Thus, the incentive compatibility constraint must bind.

Setting $r(\alpha_H) = 0$, we can obtain the corresponding interest rate, $r(\alpha_L)$, implicitly as the solution to

$$E\mathcal{W}(q, K|0, \alpha_H) = E\mathcal{W}(q, K|r(\alpha_L), \alpha_H). \tag{9}$$

We must also verify that

$$E\mathcal{W}(q, K|r(\alpha_L), \alpha_L) \geq E\mathcal{W}(q, K|0, \alpha_L). \tag{10}$$

In addition, when $\alpha = \alpha_L$, the CB must prefer the corresponding outcome to no information revelation; i.e.,

$$E\mathcal{W}(q, K|r(\alpha_L), \alpha_L) \geq E\mathcal{W}(q, K|r(\overline{\alpha}), \alpha_L), \tag{11}$$

where $r(\overline{\alpha}) = 0$. If (10) holds and π is small enough, this condition is satisfied.

Analogously to the previous section, a *symmetric equilibrium* consists of actions, (r, k), and beliefs, (μ_B, μ_i), such that (a) The CB chooses r and has beliefs μ_B such that $E^{\mu,(r,k)}[\mathcal{W}(q, k_i)|h] \geq E^{\mu,(r'',k)}[\mathcal{W}(q, k_i)|h]$, for any r', where μ_B is consistent with k; and (b) every i chooses k_i and has beliefs μ_i such that $E^{\mu,(k_i,A_{-i})}[u_i|h] \geq E^{\mu,(k_i',A_{-i})}[u_i|h]$, for any k_i', where $A_{-i} = (r, k_{-i})$ and μ_i is consistent with (k_i', A_{-i}).

The following Proposition asserts the existence of an equilibrium in which the CB truthfully signals its information. In order for the information revelation to be credible, the CB must create a monetary distortion.

Proposition 2 *Let π and ε be small and $\alpha_L = \alpha_H - \varepsilon$. Then: (a) if $\gamma > 0$, there is an equilibrium where μ_B and μ_i are consistent with $E[K \mid r, y]$, $E^{\mu,(r(\alpha_L)>0,k)}[\mathcal{W}(q, k_i)|h] > E^{\mu,(r(\alpha_L)=0,k)}[\mathcal{W}(q, k_i)|h]$, $E^{\mu,(r(\alpha_H)=0,k)}[\mathcal{W}(q, k_i)|h] > E^{\mu,(r(\alpha_H)>0,k)}[\mathcal{W}(q, k_i)|h]$ and, in either case, k is chosen such that $E^{\mu,(k_i,r,k_{-i})}[u_i|h] \geq E^{\mu,(k_i',r,k_{-i})}[u_i|h]$; (b) if $\gamma < 0$, then $E^{\mu,(r(\alpha_L)=0,k)}[\mathcal{W}(q, k_i)|h] > E^{\mu,(r(\alpha_L)>0,k)}[\mathcal{W}(q, k_i)|h]$ and $E^{\mu,(r(\alpha_H)>0,k)}[\mathcal{W}(q, k_i)|h] > E^{\mu,(r(\alpha_H)=0,k)}[\mathcal{W}(q, k_i)|h]$*

The proof is given in the Appendix. Taken together, the two Propositions imply that in order to transmit information to investors, the CB must violate the Friedman rule.[9] Incentive constraints put a lower bound on inflation in order for the information transmission to be

[9]The conditions in our two Propositions jointly hold for an open set of parameter values. Inflation rates below the Friedman rule are not consistent with the existence of a monetary equilibrium. Thus, in order to communicate its information credibly, the CB *must* choose a positive r.

credible. Otherwise, like simple announcements, investors will rationally ignore the supplied information.

4 Conclusion

We studied a model in which the CB has some information about the true state of economic fundamentals. Our main finding identifies monetary policy as a tool for credible information transmission. Inflation adds a sufficient amount of "credibility" to information provided by the CB.

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Appendix

The CB's objective can be written as:

$$
\begin{aligned}
&2E\mathcal{W}(q,k_i) = u\left(q\right) - q + F\left(\alpha_a,\alpha,y\right) = u\left(q\right) - q \\
&-\left[\frac{\left(\alpha_a y_a\right)^4 + 2\left(\alpha_a+\delta\right)\left(\alpha_a y_a\right)^2}{2\left(\alpha_a+\delta\right)^4} + \frac{1}{2\left(\alpha_a+\delta\right)^2} + \frac{3\delta^2}{2\left(\alpha_a+\delta\right)^4} + \frac{\left[3\left(\alpha_a y_a\right)^2 + \left(\alpha_a+\delta\right)\right]\delta}{\left(\alpha_a+\delta\right)^4}\right] \\
&-\frac{2\delta\left[\left(\alpha_a y_a\right)^3 + \left(\alpha_a+\delta\right)\alpha_a y_a + 3\alpha_a y_a \delta\right]}{\left(\alpha_a+\delta\right)^4} y \\
&+\left[\frac{\alpha_a + \left(\alpha_a y_a\right)^2 + 2\delta}{\left(1-\gamma\right)\left(\alpha_a+\delta\right)^2} - \frac{\left[3\delta + 3\left(\alpha_a y_a\right)^2 + \left(\alpha_a+\delta\right)\right]\delta^2}{\left(\alpha_a+\delta\right)^4}\right]\left[E_{|\alpha,y}\theta^2\right] \\
&+\left[\frac{2\delta\alpha_a y_a}{\left(1-\gamma\right)\left(\alpha_a+\delta\right)^2} - \frac{2\alpha_a y_a \delta^3}{\left(\alpha_a+\delta\right)^4}\right]\left[E_{|\alpha,y}\theta^3\right] \\
&+\left[\frac{\delta^2}{\left(1-\gamma\right)\left(\alpha_a+\delta\right)^2} - \frac{\delta^4}{2\left(\alpha_a+\delta\right)^4}\right]\left[E_{|\alpha,y}\theta^4\right], \qquad (12)
\end{aligned}
$$

Proof of Proposition 1: We have:

$$
\begin{aligned}
\frac{\partial 2E\mathcal{W}\left(\alpha_a,\alpha,y\right)}{\partial\alpha_a} = & -\frac{1}{1-\gamma}\frac{1}{\alpha^2\left(\delta+\alpha_a\right)^5}\left(6\gamma\delta^4 - 12\alpha\delta^3 + 15\alpha\gamma\delta^3 + \alpha\alpha_a^3 + 12\delta^3\alpha_a + 5\alpha\delta\alpha_a^2 + 4\alpha\delta^2\alpha_a\right. \\
& -5\alpha^2\delta\alpha_a + 3\alpha\gamma\delta^2\alpha_a + 5\alpha^2\gamma\delta\alpha_a - 10\alpha^2\delta^2 + 10\alpha^2\gamma\delta^2 + 6y^2\alpha\gamma\delta^4 - \alpha^2\alpha_a^2 + 6\delta^2\alpha_a^2 \\
& +\alpha^2\gamma\alpha_a^2 + 4y^2\alpha\delta\alpha_a^3 + 4y^2\alpha\delta^3\alpha_a + 12y^2\alpha\gamma\delta^3\alpha_a - 4y^2\alpha^2\delta^3 + 7y^2\alpha^2\gamma\delta^3 + 8y^2\alpha\delta^2\alpha_a^2 \\
& \left.+y^2\alpha^2\gamma\alpha_a^3 - 4y^2\alpha^2\delta\alpha_a^2 - 8y^2\alpha^2\delta^2\alpha_a + 6y^2\alpha\gamma\delta^2\alpha_a^2 + 9y^2\alpha^2\gamma\delta\alpha_a^2 + 15y^2\alpha^2\gamma\delta^2\alpha_a\right). \qquad (13)
\end{aligned}
$$

For $\alpha \neq 0$ and $\delta \neq 0$, this expression at $\alpha = \alpha_a$ yields:

$$
-\frac{\gamma}{1-\gamma}\frac{1}{\alpha^2\left(\delta+\alpha\right)^3}\left(3\delta\alpha + \alpha^2 + 6\delta^2 + y^2\alpha^3 + 6y^2\alpha\delta^2 + 7y^2\alpha^2\delta\right) \qquad (14)
$$

Thus, for $\gamma \neq 0$, announcing $\alpha = \alpha_a$ is not optimal. Moreover, for $\gamma \in (0,1)$, $\frac{\partial}{\partial\alpha_a}E\mathcal{W}(q,k_i)\mid_{\alpha=\alpha_a} < 0$. Thus, if the CB believes that investors will use its announcement, it announces $\alpha_a < \alpha$. If $\gamma < 0$, then $\frac{\partial}{\partial\alpha_a}E\mathcal{W}(q,k_i)\mid_{\alpha=\alpha_a} > 0$ and the CB announces $\alpha_a > \alpha$. If $\gamma = 0$, then $\frac{\partial}{\partial\alpha_a}E\mathcal{W}(q,k_i)\mid_{\alpha=\alpha_a} = 0$ and the CB announces the truth. ■

Proof of Proposition 2: Given $\gamma > 0$ (the proof for $\gamma < 0$ proceeds analogously), we show that the single crossing property is satisfied if for all y, $\frac{\partial E\mathcal{W}(\alpha_a,\alpha,y)}{\partial\alpha_a\partial\alpha} > 0$. Thus, the marginal benefit of announcing α_a is increasing in the CB's type. Notice that

$$E\mathcal{W}(\alpha_a, \alpha, y) = u(q) - q + F(\alpha_a, \alpha, y).$$

Since $u'(q) = 1 + r(\alpha_a)$, $(u(q) - q)'$ is independent of the CB's type, $\frac{\partial[u(q)-q]}{\partial\alpha_a\partial\alpha} = 0$. Thus, the single crossing property is satisfied whenever $\frac{\partial F(\alpha_a,\alpha,y)}{\partial\alpha_a\partial\alpha} > 0$. With two possible precisions, α_L and α_H, the single crossing property implies:

$$\frac{\partial F(\alpha_a, \alpha_H, y)}{\partial\alpha_a} > \frac{\partial F(\alpha_a, \alpha_L, y)}{\partial\alpha_a}. \tag{15}$$

We check this condition evaluated at $\alpha_a = \alpha_L$. The value of $\frac{\partial E\mathcal{W}(\alpha_a,\alpha,y)}{\partial\alpha_a}$, is given in (13). We have:

$$\frac{\partial E\mathcal{W}(\alpha_a, \alpha_L, y)}{\partial\alpha_a}\,|_{\alpha_a=\alpha_L} = -\frac{1}{2}\frac{\gamma}{1-\gamma}\frac{1}{\alpha_L^2(\delta+\alpha)^3}\left(3\delta\alpha_L + \alpha_L^2 + 6\delta^2 + y^2\alpha_L^3 + 6y^2\alpha_L\delta^2 + 7y^2\alpha_L^2\delta\right) < 0, \tag{16}$$

$$\begin{aligned}
\frac{\partial E\mathcal{W}(\alpha_a, \alpha_H, y)}{\partial\alpha_a}\,|_{\alpha_a=\alpha_L} = {} & -\frac{1}{2}\frac{1}{1-\gamma}\frac{1}{(\alpha_L+\varepsilon)^2(\delta+\alpha_L)^5}\left(6\gamma\delta^4 - 12(\alpha_L+\varepsilon)\delta^3 + 15(\alpha_L+\varepsilon)\gamma\delta^3\right. \\
& + (\alpha_L+\varepsilon)\alpha_L^3 + 12\delta^3\alpha_L + 5(\alpha_L+\varepsilon)\delta\alpha_L^2 + 4(\alpha_L+\varepsilon)\delta^2\alpha_L - 5(\alpha_L+\varepsilon)^2\delta\alpha_L \\
& + 3(\alpha_L+\varepsilon)\gamma\delta^2\alpha_L + 5(\alpha_L+\varepsilon)^2\gamma\delta\alpha_L - 10(\alpha_L+\varepsilon)^2\delta^2 + 10(\alpha_L+\varepsilon)^2\gamma\delta^2 \\
& + 6y^2(\alpha_L+\varepsilon)\gamma\delta^4 - (\alpha_L+\varepsilon)^2\alpha_L^2 + 6\delta^2\alpha_L^2 + (\alpha_L+\varepsilon)^2\gamma\alpha_L^2 + 4y^2(\alpha_L+\varepsilon)\delta\alpha_L^3 \\
& + 4y^2(\alpha_L+\varepsilon)\delta^3\alpha_L + 12y^2(\alpha_L+\varepsilon)\gamma\delta^3\alpha_L - 4y^2(\alpha_L+\varepsilon)^2\delta^3 + 7y^2(\alpha_L+\varepsilon)^2\gamma\delta^3 \\
& + 8y^2(\alpha_L+\varepsilon)\delta^2\alpha_L^2 + y^2(\alpha_L+\varepsilon)^2\gamma\alpha_L^3 - 4y^2(\alpha_L+\varepsilon)^2\delta\alpha_L^2 - 8y^2(\alpha_L+\varepsilon)^2\delta^2\alpha_L \\
& \left. + 6y^2(\alpha_L+\varepsilon)\gamma\delta^2\alpha_L^2 + 9y^2(\alpha_L+\varepsilon)^2\gamma\delta\alpha_L^2 + 15y^2(\alpha_L+\varepsilon)^2\gamma\delta^2\alpha_L\right),
\end{aligned} \tag{17}$$

where $\alpha_H = \alpha_L + \varepsilon$. This expression is negative for ε sufficiently small. By (15), we need to check that

$$\frac{\partial E\mathcal{W}(\alpha_a, \alpha_H, y)}{\partial\alpha_a}\,|_{\alpha_a=\alpha_L} - \frac{\partial E\mathcal{W}(\alpha_a, \alpha_L, y)}{\partial\alpha_a}\,|_{\alpha_a=\alpha_L} > 0. \tag{18}$$

This is given by:

$$\varepsilon\frac{1}{2}\frac{1}{1-\gamma}\frac{1}{\alpha_L^2\left(\delta+\alpha_L\right)^5\left(\alpha_L+\varepsilon\right)^2}\left(\alpha_L^5+6\gamma\delta^4\varepsilon+5\delta\alpha_L^4+\varepsilon\alpha_L^4+12\gamma\delta^4\alpha_L\right.$$
$$+5\delta\varepsilon\alpha_L^3+15\gamma\delta^3\varepsilon\alpha_L+16\delta^2\alpha_L^3+12\delta^3\alpha_L^2+4y^2\delta\alpha_L^5+3\gamma\delta^2\alpha_L^3+15\gamma\delta^3\alpha_L^2$$
$$+10\delta^2\varepsilon\alpha_L^2+4y^2\delta\varepsilon\alpha_L^4+3\gamma\delta^2\varepsilon\alpha_L^2+6y^2\gamma\delta^4\varepsilon\alpha_L+8y^2\delta^2\alpha_L^4+4y^2\delta^3\alpha_L^3$$
$$+6y^2\gamma\delta^2\alpha_L^4+12y^2\gamma\delta^3\alpha_L^3+6y^2\gamma\delta^4\alpha_L^2+8y^2\delta^2\varepsilon\alpha_L^3+4y^2\delta^3\varepsilon\alpha_L^2+6y^2\gamma\delta^2\varepsilon\alpha_L^3$$
$$\left.+12y^2\gamma\delta^3\varepsilon\alpha_L^2\right). \tag{19}$$

Since $\varepsilon > 0$, this expression is positive. Since truthful information is useful, it is a best response for investors to use it. ■