Monetary Emissions Trading Mechanisms*

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Abstract

Emissions trading mechanisms have been proposed, and in some cases implemented, as a tool to reduce pollution. We note that emission-trading mechanisms resemble monetary mechanisms in at least two ways. First, both attempt to implement desirable allocations under various frictions, including risk and private information. Second, in both cases implementation relies on the issue and trading of objects whose value is at least partially determined by expectations, namely (fiat) money and permits, respectively. We use insights from dynamic mechanism design in monetary economics to derive properties of *optimal dynamic emissions trading mechanisms.* We argue that efficient tax policies must be state-contingent, and we demonstrate an equivalence between such state-contingent taxes and emissions trading. Restrictions resulting from the money-like feature of permits can break this equivalence when there is endogenous progress in clean technologies. We argue that these restrictions must be taken into consideration in actual policy implementation.

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1 Introduction

Under an emissions trading system (also known as cap-and-trade), producers must acquire permits equal to the amount of their emissions in a given period. These permits are then remitted to the issuing institution.[1] So far, the results from actual implementations of emissions trading have been mixed, and some policy-makers have argued that taxes would be more effective in reducing emissions. Similar criticisms have also appeared in related academic studies. In a highly publicized recent study, Clò and Vendramin (2012) criticized features of the ETS that have led to low prices for permits. They also point out shortcomings, specifically in regard to the ability of emissions trading to induce investment in new technologies. They instead advocate a tax as a more effective non-distortionary instrument leading to price stability and increased clean investments. In a related study, Blyth, Bradley, Bunn, Clarke, Wilson, and Yang (2007) investigate how environmental policy uncertainty affects investment in low-emission technologies in the power-generation sector. In their model, firms can choose from different irreversible investments. They find that price uncertainty decreases clean investments. Chen and Tseng (2011) find that investment can be used to hedge against price risk, and it increases with uncertainty. In all these models the price of permits is treated as exogenous. Colla, Germain, and Van Steenberghe (2012) endogenize the price of the permits and study optimal policy in the presence of speculators. Finally, in a recent working paper, Albrizio and Silva (2012) introduce uncertainty over the exogenous policy rule, as well as the possibility of reversible and irreversible investments by firms. Li and Shi (2010) use a static general equilibrium model to compare regulatory emission standards and emission taxes as alternative tools for controlling emissions in a monopolistically competitive industry with heterogenous firms. They find that an emissions standard results in higher welfare than taxes if and only if productivity dispersion among firms is small and dirty firms enjoy a high degree of monopoly power.

Our analysis introduces several ingredients that are largely missing in the existing literature. First, if the policy objective is to maximize social welfare, as opposed to simply reducing emissions to a predetermined level, and if the economy is subject to shocks, then it is likely that the optimal *path* for emissions will be *time-dependent*. In particular, the welfare maximizing level of emissions will depend on the aggregate state of the economy. Second, our analysis identifies state-contingent taxes as an important tool towards implementing efficient levels of output and emissions. Third, we discuss the *optimal permit-issue policy* (similar to optimal monetary policy) in the presence of shocks.[2] We identify and explore some intriguing parallels between emission permits and fiat money. In particular, the trade value of both objects is partially determined by expectations, while their supply is set by an authority with a goal of reaching a constrained-efficient allocation for the society.

Our model is motivated by dynamic mechanism design in monetary theory, and we employ this approach to study optimal policy regarding emissions.[3] In a related influential paper, Weitzman (1974) studied price versus quantity-targeting policies in the presence of uncertainty and concluded that their effectiveness depends on the relative elasticities of supply and demand.

[1] One of the first such systems was established in the US in 1990 trough the *Clean Air Act* in order to reduce sulfur dioxide emissions. As a follow-up to the Kyoto protocol, EU countries adopted the so called *EU Emission Trading System (ETS)* in 2005 in connection to a reduction in carbon emissions.

[2] Of course, a "*central permit issuer*," an authority similar to a central bank, is not yet in existence. One implication of our analysis is to point out the need for such an authority to be established.

[3] For related applications of dynamic mechanism design to optimal taxation and to monetary theory see, for example, Golosov, Kocherlakota, and Tsyvinski (2003) and Wallace (2012).

However, Weitzman did not consider *state-contingent* policies. Following the literature on dynamic mechanism design, we will allow for state-contingent taxes in what follows. This is important as there is nothing in these models that precludes such policies and, as we will show, they tend to perform better than non-contingent ones.

We show that a state-contingent tax system can do at least as well as a cap-and-trade system in most cases, and there is a sense in which it can dominate it when there is endogenous clean technology adoption. More generally, we argue that policy-makers should think about permit-issue in a manner similar to that used by central bankers. We discuss the determination of the optimal permit-issue policy. At the optimum, the price of permits increases over time. In the absence of aggregate risk, we find that there is no role for banking. In other words, the optimum can be supported even if the permits expire at the end of the specified period of time. In the presence of aggregate risk, the optimal supply of permits is not constant over time and must respond to the shocks affecting the economy. Finally, when firms can choose the level of technological progress in green technologies, emissions trading cannot implement the optimal allocation if there is a high fraction of "dirty firms." The reason is that emissions trading either makes technology adoption by these firms too slow, or it must distort production levels relative to the first best. We show that fiscal policies do not suffer from this drawback.

2 The Model

Time is denoted by $t = 0, 1, 2, \ldots$. The economy is populated by a $[0, 1]$-continuum of firms and a $[0, 1]$-continuum of workers. Firms and workers discount the future at a rate $\beta = 1/(1 + r)$, where r is the risk free rate. There are two goods: labor and a (numeraire) good. Each firm produces the numeraire good using labor. Workers supply labor to the firm and consume the numeraire good. Using q units of labor, each firm can produce $f(q)$ units of the numeraire good. Production is costly for the society, as each operating firm creates harmful emissions. When the level of overall emissions is E, the utility of workers from consuming c units of the numeraire good and working q hours is $U(c, q, E) = u(c) - q - E$.[4] For simplicity, we assume that there is no storage across periods.

We think of emissions as being subject to random shocks, for example, due to the need to transport and use energy for cooling or heating due to weather conditions. More precisely, we assume that in each period, each firm receives a shock θ, that determines the degree of emissions generated by its production activity. At time t, the amount of emissions generated by a firm that received shock θ and that uses q units of labor is θq. For simplicity, we assume that θ is *iid* across time and across firms. We denote the cumulative distribution of θ as $G(\theta)$, with support $\left[0, \bar{\theta}\right]$.[5]

While all producing firms create pollution, they can reduce their emissions at some cost. More precisely, given θ, each firm can reduce its effective emissions to an amount y by incurring

[4] Assuming that the negative externality is generated by the flow of emissions makes our analysis readily applicable in the context of conventional pollutants such as SO_x , NO_x , Mercury, or particulates. As is well known, the stock of accumulated emissions is the relevant variable when one considers externalities related to CO_2.

[5] We make these simplifying assumptions for tractability. Assuming that emissions are proportional to the amount of input employed by the firm simplifies some of the algebra, but the results would not change if emissions were assumed to be proportional to output. The study of correlated shocks is an important topic left to future research.

the cost $h(\theta q - y)$, where $h(\cdot) : R_+ \to R_+$ is the same convex function for all firms, with $h(0) = 0$, and $h'(0) = 0$. We first study our economy in the absence of emissions control, or any other policy. In this case, firms maximize their profits without being concerned about their emissions. Since firms only differ in their degree of emissions, they behave homogeneously and they maximize their period-by-period profit. Thus, firms in each period t hire q units of labor at market wage w in order to solve

$$\pi = \max_q f(q) - wq$$

The optimal production satisfies

$$f'(q) = w \tag{1}$$

and overall emissions, E, are given by $E = q \int \theta dG(\theta)$. Taking E as given, consumers maximize their utility subject to their budget constraint. Since the numeraire good is not storable and consumers are homogeneous, there is no scope for savings. Consumers solve:

$$\begin{aligned} \max_{c,q} \quad & u(c) - q - E \\ & s.t.\ c \leq wq + \pi \end{aligned}$$

where π is the firm's profit and E is the level of total emissions. The first order conditions imply

$$wu'(c) = 1 \tag{2}$$

Finally, market clearing gives

$$c = f(q) \tag{3}$$

Combining (1) with (2) and (3) we obtain

$$f'(q)\, u'(f(q)) = 1 \tag{4}$$

We denote by $\bar{q}$ the scale of operation that solves (4). The welfare, W, in this economy is then given by

$$(1-\beta) W = u(f(\bar{q})) - \bar{q}\left(1 + \int \theta dG(\theta)\right)$$

2.1 The Efficient Allocation

Contrary to private firms, a social planner must take emissions into account when solving for the efficient outcome. It is easy to see that, since firms vary in their degree of emissions, a social planner would induce different production levels across different firms. We assume that the social planner maximizes the utility of a representative consumer:

$$\begin{aligned} \max_{q(\theta), 0 \leq y(\theta) \leq \theta q(\theta)} \quad & u(c) - \int q(\theta) + y(\theta)\, dG(\theta) \\ s.t. \quad c = & \int f(q(\theta)) - h(\theta q - y(\theta))\, dG(\theta) \end{aligned} \tag{5}$$

We denote the efficient production scale by $q^*(\theta)$ and the efficient level of emissions by $y^*(\theta)$. The schedule (q^*, y^*) satisfies the following first order conditions for all θ:

$$[f'(q^*(\theta)) - \theta h'(\theta q^*(\theta) - y^*(\theta))]\, u'(c^*) + \theta \lambda_\theta = 1 \tag{6}$$

$$h'(\theta q^*(\theta) - y^*(\theta))\, u'(c^*) - \lambda_\theta + \lambda_0 = 1 \tag{7}$$

where λ_θ is the Lagrange multiplier on $y \leq \theta q(\theta)$, and λ_0 is the multiplier on $y \geq 0$. In that case, consumption c^* is given by (5).

Not surprisingly, at the optimum, firms need to invest in order to reduce their emissions. Clearly, as $h'(0) = 0$, it is efficient to reduce emissions by a small amount for all firms.

Lemma 1 *(a) $y(\theta) < \theta q^*(\theta)$, for all θ such that $q^*(\theta) > 0$; (b) Assume $-f''(q) q / f'(q) \geq 1$, for all q. Then $\partial y^*(\theta)/\partial\theta > 0$ and there is a $\tilde{\theta} > 0$ such that $y(\theta) = 0$ for all $\theta < \tilde{\theta}$. Also, $\theta q^*(\theta) - y^*(\theta)$ is constant for all $\theta \geq \tilde{\theta}$.*

Proof. (a) Suppose there is one θ such that $\lambda_\theta > 0$, then $y(\theta) = \theta q(\theta)$. As a consequence, $\lambda_0 = 0$, and (7) implies $1 = -\lambda_\theta < 0$, which is a contradiction.

(b) Since $\lambda_\theta(\theta) = 0$, the first order conditions are

$$\begin{aligned} \left[f'(q^*(\theta)) - \theta h'(\theta q^*(\theta) - y^*(\theta))\right] u'(c^*) &= 1 \\ h'(\theta q^*(\theta) - y^*(\theta)) u'(c^*) + \lambda_0 &= 1 \end{aligned}$$

Consider first the set of θ for which $\lambda_0 = 0$. Then $y(\theta) \in (0, \theta q)$, and the first order conditions are

$$f'(q^*(\theta)) u'(c^*) = 1 + \theta \tag{8}$$

$$h'(\theta q^*(\theta) - y^*(\theta)) u'(c^*) = 1 \tag{9}$$

Given c^*, (8) implies that $q^*(\theta)$ is decreasing with θ. Also (9) implies that[6]

$$\frac{dy}{d\theta} = q + \theta \frac{dq}{d\theta} = q\left(1 + \frac{\theta}{1+\theta}\frac{f'}{f''q}\right)$$

so that $y(\theta)$ is increasing in θ if $-\frac{f''q}{f'} \geq 1$.[7] Therefore, there is $\tilde{\theta}$ such that given c^*, $\lambda_0 = 0$ and $y(\tilde{\theta}) = 0$. For $\tilde{\theta}$, $q(\tilde{\theta})$ solves

$$f'\left(q^*(\tilde{\theta})\right) u'(c^*) = 1 + \tilde{\theta}. \tag{10}$$

$$h'\left(\tilde{\theta} q^*(\tilde{\theta})\right) u'(c^*) = 1 \tag{11}$$

so that, in turn, $\tilde{\theta}$ solves

$$h'\left(\tilde{\theta}\Phi\left(\frac{1+\tilde{\theta}}{u'(c^*)}\right)\right) u'(c^*) = 1 \tag{12}$$

where $\Phi(x) = f'^{-1}(x)$. For all $\theta > \tilde{\theta}$, the solution is given by (8) and (9). Also, if $\theta < \tilde{\theta}$, it cannot be the case that $\lambda_0 = 0$. Thus, $y(\theta) = 0$, for all $\theta < \tilde{\theta}$. Notice that $\theta q^*(\theta) - y^*(\theta)$ is constant in θ whenever $y^*(\theta) > 0$; i.e., the reduction in emissions is the same for all firms. Finally, it remains to show that $\tilde{\theta} > 0$. By contradiction suppose that $\tilde{\theta} = 0$. Notice that for any $q^*(\theta)$ and $y^*(\theta)$ that satisfy (8) and (9), it must be the case that $q^*(\theta) < q^*(0)$ and $\theta q^*(\theta) \to 0$ as $\theta \to 0$. Therefore, $h'(\theta q^*(\theta) - y^*(\theta)) \to 0$ as $\theta \to 0$. Thus, for any c^* and

[6] From the last equality notice that $\theta q(\theta) - y(\theta)$ is constant. Alternatively, replace the last equality in the previous one and take the total derivative with respect to θ.

[7] This is the case, for instance, when $f(x) = \ln x$, or when $f(x) = A(1 - e^{-\alpha x})$, with $\alpha \geq 1$.

$\varepsilon > 0$, there is $\theta > 0$ such that $h'(\theta q^*(\theta) - y^*(\theta)) = \varepsilon$ and $\varepsilon u'(c^*) < 1$. This contradicts that $y(\theta) > 0$ for this θ, implying that $\tilde{\theta} > 0$. ■

Thus, all active firms ($q(\theta) > 0$) need to reduce their emissions factor at the optimum. Our assumptions also imply that, below a threshold factor, efficiency requires that firms reduce their emissions to zero. Above this threshold the optimal ex-post emissions are positive and proportional to the ex-ante emissions. The reason why $y(\theta) = 0$, for all $\theta < \tilde{\theta}$ is simple. Our specification implies that the marginal benefit of reducing emissions is the same regardless whether the reduction comes from a polluting or a non-polluting firm. The cost of emissions reduction (in terms of the loss of consumption) is small if firms are already relatively clean. This is true even if a firm eliminates its emissions entirely, as $h'(\theta q^*(\theta))$ converges to zero when θ is small. Hence, the optimal total emissions level is given by $E^* = \int y^*(\theta)\, dG(\theta)$. Interestingly, the efficient allocation dictates that some firms reduce their emissions, by *both* reducing their production scale *and* by cleaning their act. Of course, in the absence of taxes or other emission control policies, all firms operate at the same scale and none becomes cleaner.

For later reference, it is instructive to consider the following thought experiment. Consider two economies which are identical except that one is subject to a θ-distribution G_0, while the other is subject to distribution G_1, where $\int \theta dG_1(\theta) < \int \theta dG_0(\theta)$. In words, firms are on average cleaner in the economy under G_1. Comparing the efficient allocations in the two economies gives us the following.

Lemma 2 *The optimal allocations are such that $\tilde{\theta}_1 > \tilde{\theta}_0$. For all $\theta > \tilde{\theta}_1$, $q_1^*(\theta) < q_0^*(\theta)$ and $y_1^*(\theta) < y_0^*(\theta)$.*

Proof. First, notice from (8) and (9) that given a level of aggregate consumption c^*, the efficient production, $q^*(\theta)$, is decreasing in θ whenever $y(\theta) > 0$. Since there is a larger fraction of relatively clean firms in the economy with G_1 (while the mass of firms is the same), we can infer that $c_1^* > c_0^*$. In this case, (8) gives us $f'(q_1^*)\, u'(c_1^*) = f'(q_0^*)\, u'(c_0^*)$ whenever $y_1^*, y_0^* > 0$. Therefore, $q_1^*(\theta) < q_0^*(\theta)$; i.e., firms with the same θ produce relatively less in the cleaner economy. Finally, from (9), $h'(\theta q_1^*(\theta) - y_1^*(\theta))\, u'(c_1^*) = h'(\theta q_0^*(\theta) - y_0^*(\theta))\, u'(c_0^*)$, so that $\theta q_1^*(\theta) - y_1^*(\theta) > \theta q_0^*(\theta) - y_0^*(\theta)$, and firms with the same θ reduce their emissions more in the cleaner economy. Next, we demonstrate that $\tilde{\theta}_1 > \tilde{\theta}_0$. First, notice that, for any c, $\theta q(\theta)$ is increasing in θ if $-f''q \geq f'$. This implies that, given c, $\theta \Phi(\frac{1+\theta}{u'(c)})$ is increasing in θ, where $\Phi(x) = f'^{-1}(x)$. Second, since $\Phi'(x) < 0$ and $u'(c_1^*) < u'(c_0^*)$, we must have that $\theta \Phi(\frac{1+\theta}{u'(c_1^*)}) < \theta \Phi(\frac{1+\theta}{u'(c_0^*)})$, for any θ. However, (12) implies that $\tilde{\theta}_1 \Phi(\frac{1+\tilde{\theta}_1}{u'(c_1^*)}) > \tilde{\theta}_0 \Phi(\frac{1+\tilde{\theta}_0}{u'(c_0^*)})$. Since $\theta \Phi(\frac{1+\theta}{u'(c)})$ is increasing in θ, it implies that $\tilde{\theta}_1 > \tilde{\theta}_0$. Therefore, more firms are clean ex-post in the economy with G_1. ■

Typically, efficiency will require a reduction in emissions from their level under laissez-faire. One possible tool towards accomplishing this involves imposing a tax. Another possibility, which we study first, involves imposing controls over emissions, together with a market for emissions permits, so that firms which pollute most internalize the cost of their emissions.

3 Policy

We first consider an economy where firms participate in a market for permits.

3.1 Emissions Trading

We assume that if a firm produces q units of goods, and given its emission shock is θ, it will need to accumulate θq units of emission permits. Alternatively, a firm might invest in order to reduce its pollution to ex-post emission level $y(\theta) \leq \theta q$ and then accumulate $y(\theta)$ units of permits. The permits are then remitted once production takes place. There is a market where firms can trade permits. The (equilibrium) price of permits in terms of the numeraire will be denoted by ϕ. The sequence of events is as follows:

1. Firms receive their shock θ and plan to produce q
2. Firms reduce their emissions factor to $y(\theta) \leq \theta q$
3. Firms produce and enjoy profit $f(q) - wq - h(\theta q - y(\theta))$
4. Firms adjust their permits in the market and remit $y(\theta)$ permits
5. Profit, if any, is redistributed to shareholders
6. Firms begin the next period

We assume that the total stock of *"em"-ission* permits in this economy is M and we define the firm's problem recursively. A firm's individual holdings of permits are denoted by m. We denote the value function of a firm entering the market with m permits and a shock θ by $V(m;\theta)$. This value is defined by

$$\begin{aligned} V(m;\theta) &= \max_{q,y,m_+} f(q) - wq - h(\theta q - y) + \phi(m - y - m_+) + \beta E_\theta V(m_+ + T;\theta) \\ s.t.\ 0 &\leq y \leq \theta q \end{aligned}$$

where T is a transfer of permits by the issuing authority. When the firm enters the market for permits, the value of its portfolio is ϕm. The firm then has to remit y permits (with value ϕy) and decides on how many permits to carry over to the next period, m_+. As a consequence, the firm's profit changes by the amount $\phi(m - y - m_+)$. Given M, the market clearing conditions are

$$\int y(\theta) + m_+(\theta)\, dG(\theta) = M \tag{13}$$

$$\int f(q(\theta)) - h(\theta q(\theta) - y)\, dG(\theta) = c \tag{14}$$

The law of motion for the stock of permits is

$$M_+ = M - \int y(\theta)\, dG(\theta) + T$$

Given a policy $\{T_t\}$, an equilibrium is a list of prices, $\{\phi_t\}$, a list of quantities and emissions, $\{c_t, q_t(\theta), y_t(\theta)\}$, and trading decisions, $\{m_t(\theta)\}$, such that, given prices, the decision variables solves the firms' and consumers' problem and markets clear. An equilibrium is stationary whenever the list of quantities and emissions is time independent; i.e., if $\{c_t, q_t(\theta), y_t(\theta)\} =$

$\{c, q(\theta), y(\theta)\}$, for all t. Next, we demonstrate that there is a unique stationary equilibrium. We first solve the firm's problem. The first order conditions give

$$\begin{aligned} f'(q) - \theta h'(\theta q - y) &= w - \theta\phi\lambda_\theta(\theta) &(15)\\ h'(\theta q - y) - \phi - \phi\lambda_\theta(\theta) + \phi\lambda_0(\theta) &= 0 &(16)\\ \beta E_\theta V_m(m_+ + T; \theta) &\leq \phi = \text{if } m_+ > 0 &(17)\end{aligned}$$

where $\phi\lambda_\theta(\theta)$, $\phi\lambda_0(\theta)$ are the multipliers on the firm's constraints. Notice that all firms will exit the market for permits with the same amount of permits for the next period. The envelope condition gives

$$V_m(m; \theta) = \phi \tag{18}$$

and using this expression in (17) we obtain that $\beta E_\theta \phi_+ \leq \phi$, with equality if firms carry permits from one period to the next. As ϕ_+ does not depend on the *i.i.d.* idiosyncratic shock θ, this gives us

$$\beta\phi_+ \leq \phi \ (= \text{ if } m_+ > 0) \tag{19}$$

In words, firms are willing to hold permits if the appropriately discounted futures price for permits equals today's spot price. If today's spot price is higher, then firms prefer to buy their permits tomorrow, and no permits are held across periods. This will be the case if the issuing authority is supplying enough permits in the market tomorrow. However, there is no equilibrium if today's spot price is lower, as firms will try to purchase an infinite amount of permits today to resell in tomorrow's futures market.

Like before, the worker's decision is given by (2), and, using market clearing, we obtain an expression for the wage.

$$wu'\left(\int f(q(\theta)) - h(\theta q(\theta) - y(\theta))\, dG(\theta)\right) = 1. \tag{20}$$

To solve for $y(\theta)$, first notice that all firms will reduce their ex-post emissions whenever permits are costly to acquire. Formally, we have the following.

Lemma 3 *$y(\theta) < \theta q$, for all θ, whenever $\phi > 0$.*

Proof. $y(\theta) < \theta q$, for all θ, implies that $\lambda_\theta(\theta) = 0$, for all θ. Indeed, suppose there is one θ such that $\lambda_\theta(\theta) > 0$ and $y(\theta) = \theta q$. Then $\lambda_0(\theta) = 0$ and since $h'(0) = 0$, (16) gives us

$$-\phi - \phi\lambda_\theta(\theta) = 0$$

which is impossible when $\phi > 0$. ■

The following Lemma states that relatively clean firms do not emit any ex-post emissions if permits are costly to acquire. The more costly permits are, the more firms choose not to pollute ex-post. In addition, the choice of production level in equilibrium does not depend on the price of permits, but only on the realized marginal cost of emissions.

Lemma 4 *Suppose $\phi > 0$. Then there is $\bar{\theta}(\phi) > 0$ such that for all $\theta \leq \bar{\theta}(\phi)$, we have that $y(\theta) = 0$. The quantity produced, $q(\theta, w)$, is decreasing in θ and w. In addition, $\bar{\theta}'(\phi) > 0$.*

Proof. Let us consider the case of a firm that does not emit any emissions ex-post; i.e., $y(\theta) = 0$, for some θ. In this case, $\lambda_0(\theta) > \lambda_\theta(\theta) = 0$ and the firm's solution is

$$f'(q) - \theta h'(\theta q) = w \tag{21}$$

$$h'(\theta q) \leq \phi \tag{22}$$

Notice that the LHS of (21) is strictly decreasing in q, so that (21) defines a function $q(\theta)$ that is uniquely defined for each θ. It is easy to check that $q'(\theta) < 0$. In addition, $q(\theta)$ is decreasing in w for all θ such that $y(\theta) = 0$. Finally, notice that the LHS of (22) is increasing in θ: taking the total derivative, and using the expression for $q'(\theta)$ from (21), we obtain[8]

$$\frac{dh'(\theta q)}{d\theta} = q\left(1 - \frac{\theta^2 h''}{\theta^2 h'' - f''}\right) h'' > 0 \tag{23}$$

where the inequality follows from the concavity of the production function. Therefore, there is a $\bar{\theta}$ such that $y(\theta) = 0$, for all $\theta < \bar{\theta}$. The threshold $\bar{\theta}$ is defined by

$$h'\left(\bar{\theta} q\left(\bar{\theta}\right)\right) = \phi \tag{24}$$

Whenever $\lambda_\theta(\theta) = 0$, the emission constraint is not binding, and, from (21), $q(\theta)$ is not an explicit function of ϕ. Therefore, when ϕ increases, $\bar{\theta}$ also has to increase by (23): more firms choose to reduce their emissions to a full extent when the price of permits increases. ■

Note one effect of general equilibrium analysis. The solution $q(\theta)$ to (21) does not necessarily coincide with the efficient level $q^*(\theta)$. Indeed, notice that the wage is given by (20). If a positive measure of firms do not follow the social planner's production plan, the wage is distorted and so is the decision of firms with $y(\theta) = 0$.

For relatively high-polluting firms, we obtain the following characterization. Dirtier firms reduce emissions by the same amount; i.e., the difference between ex-ante and ex-post emissions is the same. Dirtier firms have higher ex-post emissions, but ex-post emissions decline as permits become more expensive to acquire. The production of dirtier firms is declining in the wage, their degree of dirtiness, θ, and in the price of permits.

Lemma 5 *Suppose $\phi > 0$. Then, for all $\theta > \bar{\theta}(\phi)$, we have that $y(\theta, \phi)$ and $q(\theta, \phi, w)$ are such that $0 < y < \theta q$, $\theta q - y$ is a constant function of ϕ, $y_1(\theta, \phi) > 0$, $y_2(\theta, \phi) < 0$, and $q_i(\theta, \phi, w) < 0$, for $i = 1, 2, 3$.*

Proof. Let us consider the case when $0 < y(\theta) < \theta q$. Setting $\lambda_\theta(\theta) = \lambda_0(\theta) = 0$, the solution of the firm becomes,

$$f'(q) - \theta h'(\theta q - y) = w \tag{25}$$

$$h'(\theta q - y) = \phi \tag{26}$$

[8] From (21), we have

$$\left(f'' - \theta^2 h''\right) q'(\theta) = \theta q h''.$$

Therefore,

$$\frac{dh'}{d\theta} = \theta h'' q'(\theta) + q h'' = h''\left(\theta q'(\theta) + q\right) = h''\left(\theta \frac{\theta q h''}{f'' - \theta^2 h''} + q\right) = q h''\left(1 - \frac{\theta^2 h''}{\theta^2 h'' - f''}\right).$$

Replacing the expression for h' in the first equation, we obtain

$$\begin{aligned} f'(q(\theta)) &= w + \theta\phi && (27) \\ h'(\theta q(\theta) - y(\theta)) &= \phi && (28) \end{aligned}$$

For firms with $\theta > \bar{\theta}$, the solution is a pair $(q(\theta), y(\theta))$ that solves (27) and (28) jointly. Notice that $q'(\theta) < 0$ whenever $\phi > 0$. Also, using (28) and the expression for $q'(\theta)$, we obtain that $y'(\theta) > 0$ if $-f''(q)\, q/f'(q) \geq 1$. Finally, if ϕ increases, (27) implies that $q(\theta)$ declines, in which case (28) implies that $y(\theta)$ is also decreasing in ϕ. ■

Interestingly, the higher the price of permits, the lower the wage. Since permits are more costly to acquire, more firms decide to spend resources to reduce their ex-ante emissions. Those firms who still emit ex-post also reduce their production scale. Therefore, they do not employ as much labor as when the price of permits is low. As a result, the wage has to fall. In other words, we have the following.

Lemma 6 $w'(\phi) < 0$.

Proof. Given firms' optimal behavior, the wage is given by

$$wu'\left(\int_0^{\bar{\theta}} f(q(\theta)) - h(\theta q(\theta))\, dG(\theta) + \int_{\bar{\theta}}^{\infty} f(q(\theta)) - h(\theta q(\theta) - y(\theta))\, dG(\theta)\right) = 1. \tag{29}$$

Since $q(\theta)$ does not depend on ϕ whenever $\theta < \bar{\theta}$, we obtain

$$u'\frac{\partial w}{\partial \phi} + wu''\left(\int_{\bar{\theta}}^{\infty} w\frac{\partial q}{\partial \phi} + p\frac{\partial y}{\partial \phi} dG(\theta)\right) = 0 \tag{30}$$

where we have used (27) and (28). Since $\frac{\partial q}{\partial \phi} < 0$ and $\frac{\partial y}{\partial \phi} < 0$, we have $\frac{\partial w}{\partial \phi} < 0$. When studying the general equilibrium effect of a rise in ϕ, it is important to notice that the effect of the rise in ϕ on $q(\theta)$ is somewhat tempered by the decline in wage w. Therefore, a part of the effect of the rise in ϕ on the use of input is absorbed by the general decline in the wage. Still, $q(\theta)$ and $y(\theta)$ are decreasing functions of ϕ, even when considering the general equilibrium effects. ■

The equilibrium price level is a function of the policy, T. As expected, if there is a high volume of permits in circulation, they have no market value.

Lemma 7 *Suppose $T \geq \bar{q} \int \theta dG(\theta)$. Then $\phi = 0$ and $q(\theta) = \bar{q}$, for all θ.*

Proof. Since $T \geq \bar{q} \int \theta dG(\theta)$, we have that $M \geq \bar{q} \int \theta dG(\theta)$ in any period. We first guess that $m_+ = 0$ and show that this is consistent with equilibrium. Denote by $y(\theta, \phi)$, the choice of emissions by firm θ, given that the price of permits is ϕ. From the market clearing condition for permits (13), using $m_+ = 0$, we have

$$M = \int y(\theta, \phi)\, dG(\theta) \tag{31}$$

We have shown that emissions $y(\theta, \phi)$ are a decreasing function of ϕ, for all θ. Thus, $y(\theta, \phi) \leq y(\theta, 0) = \theta\bar{q}$. But since $M \geq \bar{q} \int \theta dG(\theta)$, (31) cannot hold. Hence, the only equilibrium is when $\phi = 0$ and $q(\theta) = \bar{q}$. (19) then implies that $\phi_+ = 0$, which is consistent with $m_+ = 0$. ■

Notice that firms receive a transfer of new permits, T, in each period, so that they are not forced to carry permits from one period to the next. Also notice from (31) that one way to achieve the efficient level of production, $q^*(\theta)$, is to set M and T such that $M = T = \int y^*(\theta)\, dG(\theta) = E^*$, so that the stock of permits is just sufficient to cover the efficient amount of emissions, E^*. In this case, the unique equilibrium price, ϕ, is $\phi = 1/u'(c^*)$, and $m_+ = 0$, as $\beta\phi_+ < \phi$. Thus, there is no banking of permits. In our stationary economy, where the distribution of emissions is the same in each period, this implies that the stock of permits should be set at E^*. This discussion is summarized in the following.

Proposition 8 *The equilibrium with permits is efficient if $M = T = E^*$ for all t. The banking of permits is not necessary for efficiency.*

Proof. Using (65) and the fact that $\lambda_\theta(\theta) = 0$, for all θ, the firm's first order condition can be re-arranged as

$$[f'(q) - \theta h'(\theta q - y)]\, u'(c) = 1 \quad (32)$$
$$h'(\theta q - y) = \phi\,[1 - \lambda_0(\theta)] \quad (33)$$

Setting $M = E^*$ the equilibrium is $y = y^*(\theta)$, $q = q^*(\theta)$, and ϕ satisfies

$$\phi u'(c^*) = 1.$$

Indeed, given this ϕ, we can define $\lambda_0(\theta) = \lambda_0$, where λ_0 is the multiplier in (7). Then the firm's FOC and the planner's FOC coincide. Therefore, $M = E^*$ implements the efficient allocation.[9]
■

3.2 Taxes

In this subsection we investigate the implications of taxing emissions. We assume that, while the government does not observe θ, a firm's emissions level, $y(\theta)$, is verifiable, so the government can impose a tax, τ, on emissions once production takes place. For simplicity, we assume the tax schedule is history-independent, so that $\tau_t(e|h_t) = \tau_{t+1}(e|h_{t+1})$, where h_t is the history of emissions up to and including date $t-1$. The tax proceeds are then distributed to consumers as a lump-sum transfer. The firm's problem is essentially static. At the start of a period, a firm which received shock θ solves the following:

$$\max_{q,y} f(q) - wq - h(\theta q - y) - \tau(y)$$
$$s.t.\ 0 \leq y \leq \theta q$$

The first order conditions are

$$f'(q) - w - \theta h'(\theta q - y) = 0$$
$$h'(\theta q - y) - \tau'(y) + \tilde{\lambda}_0 - \tilde{\lambda}_\theta = 0$$

[9] In the Appendix we show that the structure of the equilibrium does not change if the issuing authority sells permits instead of simply assignining them as transfers. These two methods are essentially the same for our purposes.

The consumer's problem is

$$\max u(c) - q$$
$$s.t.\ c \leq wq + T$$

where T is the government's lump sum transfer. The first order conditions remain the same as before:

$$wu'(c) = 1 \tag{34}$$

The planner's first order conditions are

$$[f'(q(\theta)) - \theta h'(\theta q - y)]\, u'(c^*) = 1 \tag{35}$$
$$h'(\theta q - y(\theta))\, u'(c^*) + \lambda_0 - \lambda_\theta = 1 \tag{36}$$

It is then easy to see the following.

Proposition 9 *The tax schedule $\tau(e) = \frac{e}{u'(c^*)}$ implements the efficient allocation.*

3.3 Aggregate Risk

So far we have assumed that there is no aggregate risk. As a result, the optimal level of emissions and consumption are known. Here we consider the case where the function G is random. In that case, c^* will be a function of G, which is not observable. Yet, we will argue that both cap and trade and a *state-contingent* tax can support the efficient levels of consumption and emissions in our economy.

To see this, consider the case where emissions are drawn from a new distribution G_1 instead of the initial distribution G_0, where $\int \theta dG_1(\theta) < \int \theta dG_0(\theta)$. In words, firms are on average cleaner and, as a result, E^* decreases, from E_0^* to $E_1^* < E_0^*$. Clearly, any tax system which does not depend on any aggregate variable, will not achieve the first best. Let us consider a tax system that is measurable with respect to all aggregate variables at the time of production. There is only one variable that is observable at the time of production and that is the wage level, w. Then a firm that emits y has to pay the government $\tau(e; w)$. Given G_i, let c_i^* be the planner's solution for consumption and w_i such that $w_i u'(c_i^*) = 1$. Then we can define the tax schedule $\tau(e; w)$ as follows:

$$\tau(e; w_i) = \frac{e}{u'(c_i^*)} = ew_i$$

The same analysis as before shows that this tax schedule implements the efficient allocation.

We now turn to the cap-and-trade system. We start from the old steady state with optimal policy $M = E_0^*$. We demonstrate that if $M = E_0^*$, the new steady state, where $\theta \sim G_1$ will be characterized by a lower price of permits, $\phi_1 < \phi_0$. This is true if $E_0^* > \bar{q}_1 \int \theta dG_1(\theta)$. Now, consider the case where $\bar{q}_1 \int \theta dG_1(\theta) > E_0^*$. The firms' decisions are still given by (15)-(17). In particular, if $\phi_1 > 0$, we still have that $\lambda_\theta(\theta) = 0$, for all θ (it is still optimal to reduce emissions by a tiny amount), so that the first order conditions become

$$f'(q) - \theta h'(\theta q - y) = w \tag{37}$$
$$h'(\theta q - y) + \phi_1 \lambda_0(\theta) = \phi_1 \tag{38}$$
$$\beta E_\theta V_{m_+}(m_+; \theta) \leq \phi_1 \tag{39}$$

Now suppose, by way of reaching a contradiction, that $\phi_1 \geq \phi_0$. Since $m_+(\phi_0) = 0$, we also have $m_+(\phi_1) = 0$. From the market clearing conditions (with $m_+ = 0$), we obtain

$$\int y(\theta, \phi_1)\, dG_1(\theta) = E_0^*. \tag{40}$$

Also, as the FOCs remain the same, $y(\theta, \phi)$ still has the same properties as before: it is increasing in θ and decreasing in ϕ. Everything else is constant and we already showed that $y'(\theta) > 0$. Therefore,

$$\int y(\theta, \phi_0)\, dG_1(\theta) < \int y(\theta, \phi_0)\, dG_0(\theta)$$

We have shown that $y(\theta, \phi)$ is a decreasing function of ϕ for all θ, so that

$$\int y(\theta, \phi_1)\, dG_1(\theta) < \int y(\theta, \phi_0)\, dG_1(\theta) < \int y(\theta, \phi_0)\, dG_0(\theta) = E_0^*$$

However, this violates the equilibrium condition (40). Hence, we must have $\phi_0 > \phi_1$.

In summary, both taxes and emission trading can support the efficient allocation. This conclusion relies on considering state-contingent taxes. While such taxes are not typically studied in the literature, there is nothing in the economic environment that precludes their use. In the case of cap and trade, the market price for permits acts as a signalling device. It declines because firms are on average cleaner. Notice, however, that without an exogenous change in the supply of permits, total emissions E will remain constant, and will diverge from the efficient level of emissions. This calls for an authority that can manage the stock of permits so as to keep the price at ϕ_0. Our analysis recommends that the price of permits should be a policy variable for this authority, very much like the supply in the money market is controlled by a central bank.

4 Endogenous Technological Change

So far we found that state-contingent taxes and emissions trading are equally successful in supporting efficient outcomes. Our analysis has abstracted from issues related to technological change. These issues are important, and it would be interesting to know if one policy dominates if the possibility of endogenous technological change is introduced. In this section we extend our environment to account for this possibility.

Like before, we identify firms by their type, θ, regarding their tendency to pollute. Here we assume that types are distributed at $t = 0$ according to the cumulative distribution G with support $[0, \bar{\theta}]$. Like before, a θ-firm emits θq units of pollution whenever it uses q units of labor. We will refer to θ as the technological emissions factor. We will assume that firms can hire labor in order to invent/adopt new, cleaner technologies. To capture the fact that returns to R&D involve an element of randomness, we assume that by devoting γ units of labor, a firm can enter the following lottery. If a θ-firm pays this cost, it receives the new emission factor $\tilde{\theta} = 0$ with probability s in the next period. With probability $1 - s$, its emission factor is the same as before, $\tilde{\theta} = \theta$. In words, with probability s a firm becomes clean forever and with probability $1 - s$ it remains as dirty as before. Other than this feature, the model remains the same as in the previous sections.[10]

[10]Note that this specification results in a non-stationary equilibrium fraction of clean firms.

We will consider the simplest case, where $G(\theta)$ has a two point support, $\{0,\bar{\theta}\}$, with $G(0)=\mu$ denoting the mass of clean firms. Conveniently, the distribution of firms in every period is summarized by the mass of clean firms, which greatly simplifies the analysis. The social planner chooses non-negative consumption, c, production, $q(\theta)$, and a choice of new technology adoption $i(\theta)\in[0,1]$ for each firm. Clearly, the planner would not invest in a new technology for clean firms, so we let $i\left(\bar{\theta}\right)\in[0,1]$ be the mass of dirty firms entering the lottery. Given that there is a need for γ units of labor to enter the lottery, the consumer's utility function is reduced by the amount of labor devoted to research and development $(1-\mu)\gamma i$.[11] We denote by V the objective function of the planner given an initial distribution μ. To reduce notation, in what follows we use $i=i\left(\bar{\theta}\right)$, $q=q(0)$, $\bar{q}=q\left(\bar{\theta}\right)$, while $\mu_{+}=\mu+(1-\mu)\,si\left(\bar{\theta}\right)$ denotes the measure of clean firms in the next period. The planner solves the following problem

$$\begin{aligned}
V(\mu) &= \underset{c,q,\bar{q},i}{Max}\; u(c)-\mu q-(1-\mu)\left(1+\bar{\theta}\right)\bar{q}-(1-\mu)\gamma i+\beta V(\mu+(1-\mu)\,si) \\
s.t.\;\; c &= \mu f(q)+(1-\mu)\,f(\bar{q}) \\
0 &\leq i\leq 1
\end{aligned}$$

Given the linearity of the objective function in i, we can obtain an explicit form for $V(\mu)$. Notice first that the solution for c, q, and $\bar{q}$ does not depend on i. Replacing the market clearing condition in the planner's objective and taking the first order conditions with respect to q and $\bar{q}$, we obtain

$$u'\left(\mu f(q)+(1-\mu)\,f(\bar{q})\right)f'(q) = 1 \tag{41}$$

$$u'\left(\mu f(q)+(1-\mu)\,f(\bar{q})\right)f'(\bar{q}) = 1+\bar{\theta} \tag{42}$$

Given μ, (41) and (42), define the solution by $q^{*}(\mu)$ and $\bar{q}^{*}(\mu)$, independently of i. Plugging these values in the market clearing condition gives us $c^{*}(\mu)$. Thus, the planner's problem becomes

$$\begin{aligned}
V(\mu) &= \max_{i}\; F(\mu)-(1-\mu)\gamma i+\beta V(\mu+(1-\mu)\,si) \\
s.t.\;\; 0 &\leq i\leq 1
\end{aligned}$$

where $F(\mu)\equiv u(c^{*}(\mu))-\mu q^{*}(\mu)-(1-\mu)\left(1+\bar{\theta}\right)\bar{q}^{*}(\mu)$. As the solution to (41) and (42) is unique, there is a single value of μ such that $F(\mu)=v$, for each value of the instant surplus v. Also, F is differentiable with

$$F'(\mu)=u'^{*}(\mu))[f(q^{*}(\mu))-f(\bar{q}^{*}(\mu))]-q^{*}(\mu)+(1-\theta)\bar{q}^{*}(\mu)$$

and $F''(\mu)=u''^{*}(\mu))[f(q^{*}(\mu))-f(\bar{q}^{*}(\mu))]c'(\mu)<0$. Our assumptions on preferences and technology guarantee that F'^{-1} exists. Let $\Phi\equiv F'^{-1}\left(\frac{1-\beta}{\beta}\frac{\gamma}{s}\right)$. We now guess that the value function takes the form

$$V(\mu)=F(\mu)+\frac{\gamma}{s}(\mu-\Phi)+\frac{\beta}{1-\beta}F(\Phi) \tag{43}$$

To verify, using (43) the planner's problem becomes

$$\begin{aligned}
&\max_{i}\; F(\mu)-(1-\mu)\gamma i+\beta\left[F(\mu+(1-\mu)\,si)+\frac{\gamma}{s}(\mu+(1-\mu)\,si)-\frac{\gamma}{s}\Phi+\frac{\beta}{1-\beta}F(\Phi)\right] \\
&s.t.\;\; 0\leq i\leq 1
\end{aligned}$$

[11] We assume that R&D itself is not a polluting activity.

Assuming an interior solution, the first order condition gives

$$-\gamma + \beta s F'\left(\mu + (1-\mu) si\right) + \beta\gamma = 0$$

or

$$i = \frac{\Phi - \mu}{(1-\mu) s} \tag{44}$$

Using this policy function in the objective function, we obtain

$$\begin{aligned} V(\mu) &= F(\mu) - (1-\mu)\gamma \frac{\Phi-\mu}{(1-\mu)s} + \beta\left[F(\Phi) + \frac{\gamma}{s}\Phi - \frac{\gamma}{s}\Phi + \frac{\beta}{1-\beta}F(\Phi)\right] \\ &= F(\mu) - \frac{\gamma}{s}(\Phi-\mu) + \beta\left[F(\Phi) + \frac{\beta}{1-\beta}F(\Phi)\right] \\ &= F(\mu) + \frac{\gamma}{s}(\mu-\Phi) + \frac{\beta}{1-\beta}F(\Phi) \end{aligned}$$

which verifies our guess (43). Notice that Φ is a constant in $[0,1]$ and (44) gives us $\frac{\partial i}{\partial \mu} = \frac{-(1-\mu)s+(\Phi-\mu)s}{[(1-\mu)s]^2} < 0$. Hence, as the measure of clean firms increases, the planner reduces investment in the clean technology. Clearly, there is a $\bar{\mu}$ such that for all $\mu \geq \bar{\mu}$, the planner chooses $i(\bar{\mu}) = 0$. The threshold level, $\bar{\mu}$, is defined by

$$\begin{aligned} \Phi &= \bar{\mu}, \text{ or} \\ F'(\bar{\mu}) &= \frac{1-\beta}{\beta}\frac{\gamma}{s} \end{aligned} \tag{45}$$

If there is no emissions control, firms maximize their profits without being concerned about their emissions and their production decision follows (1). No firm would invest in emissions reduction, as the investment in R&D is costly. Since firms' production decision is independent of their shock, overall emissions, E, capture the emissions from dirty firms, or $\bar{E} = (1-\mu)\bar{\theta}\bar{q}$, where $\bar{q}$ is the equilibrium level of production. Taking E as given, consumers maximize their utility subject to their budget constraint and their behavior is again summarized by the first order condition (2). Market clearing is given by (3) and the equilibrium level of production $\bar{q}$ satisfies (4); i.e., $f'(q)\,u'(f(q)) = 1$. Welfare in this economy is given by

$$(1-\beta)\,W = u\left(f(\bar{q})\right) - \bar{q}\left(1 + (1-\mu)\bar{\theta}\right)$$

4.1 Emissions Trading

We now consider an economy where firms are subject to a cap and trade system: a dirty firm producing q units of goods and receiving emission factor $\bar{\theta}$, will need to accumulate $\bar{\theta}q$ permits. The permits are then remitted once production takes place. As before, firms can also invest in order to reduce their emissions. There is a market where firms can trade permits. The price of permits in terms of the numeraire is again denoted by ϕ. The sequence of events is as follows:

1. Firms of type $\theta \in \{0, \bar{\theta}\}$ plan to produce $q(\theta)$ and invest $i(\theta)$ in clean technologies. We assume that firms are able to randomize, so $i(\theta) \in [0,1]$ denotes the probability of investing in clean technology R&D

2. Firms produce and enjoy profit $f(q) - w(q + \gamma I)$, where w is the wage and $I \in \{0, 1\}$ is the result of the lottery $i(\theta)$

3. Firms adjust their permits in the market and remit θq permits

4. Profit, if any, is redistributed to shareholders

5. Firms learn the result of their R&D investment and move on to the next period

Like before, we denote the total stock of permits in this economy by M, while a firm's individual permit holdings are denoted by m. We denote the value of a dirty firm entering the futures market with m permits and shock $\bar{\theta}$ by $V_{\bar{\theta}}(m)$, and the value for a corresponding clean firm by $V_0(m)$. Hence, $V_\theta(m)$ for $\theta \in \{0, \bar{\theta}\}$ is defined by

$$\begin{aligned} V_\theta(m) &= \max_{q,i,m_+^0,m_+^\theta} f(q) - w(q + \gamma i) + \phi m - \phi\theta q \\ &\quad + is\left[-\phi m_+^0 + \beta V_0\left(m_+^0 + T_+^0\right)\right] + \left[i(1-s) + (1-i)\right]\left[-\phi m_+^\theta + \beta V_\theta(m_+^\theta + T_+^\theta)\right] \\ s.t.\ 0 &\leq i \leq 1 \end{aligned}$$

where T_+^θ is the (emission factor-specific) transfer of permits by the issuing authority. When the firm enters the market for permits, the value of its portfolio is ϕm. The firm then has to remit θq permits with value $\phi\theta q$ and decides on how many permits to carry over to the next period, m_+. The first order conditions for an interior condition $i(\theta) \in (0, 1)$ are

$$f'^{\theta}) - w - \phi\theta = 0 \quad (46)$$

$$-w\gamma + s\left[-\phi m_+^0 + \beta V_0\left(m_+^0 + T^0\right)\right] - s\left[-\phi m_+^\theta + \beta V_\theta(m_+^\theta + T_+^\theta)\right] \leq 0 \quad (47)$$

$$(= \text{ if } i > 0, \ > 0, \text{ if } i = 1)$$

$$-\phi + \beta V_0'(m_+^0 + T_+^0) \leq 0 (= \text{if } m_+^0 > 0)$$

$$-\phi + \beta V_\theta'(m_+^\theta + T_+^\theta) \leq 0 (= \text{if } m_+^\theta > 0)$$

and the envelope condition gives $V_\theta'(m) = \phi$, for $\theta \in \{0, \bar{\theta}\}$. The first order condition for $i(0)$ clearly implies that $i(0) = 0$, as clean firms remain clean. The last two conditions imply that in an equilibrium with banking (in which firms carry permits from one period to the next) the price of permits must satisfy

$$\phi = \beta\phi_+$$

The consumer's first order conditions give

$$wu'(c) = 1 \quad (48)$$

Finally, market clearing implies

$$\begin{aligned} \mu f\left(q^0\right) + (1-\mu) f\left(q^{\bar{\theta}}\right) &= c \quad (49) \\ \mu m_+^0 + (1-\mu) m_+^1 &= M + T \end{aligned}$$

and the law of motion for clean firms is $\mu_+ = \mu + si(\bar{\theta})(1-\mu)$. Next, we determine whether the efficient allocation is implementable. We divide the analysis into three cases. First we discuss the policy on permits which implements the efficient allocation when $\mu \geq \bar{\mu}$. Second, we consider the case where $\mu < \bar{\mu}$ but close to μ. Finally, we consider the case where μ is far below $\bar{\mu}$.

(i) Case when $\mu \geq \bar{\mu}$

First, note that the equilibrium outcome in an economy with banking is inefficient for all $\mu \geq \bar{\mu}$. Indeed, in this case the efficient allocation is such that $q(\bar{\theta})$ satisfies $wf'(\bar{q}) = 1 + \bar{\theta}$, where $w = u'$ is a constant. But this can only be the case if $\phi = w$, a constant. Therefore $\phi > \beta\phi_+ = \beta\phi$. This contradicts the efficiency of banking. The only other way to reach the efficient allocation when $\mu \geq \overline{\mu}$ is through a transfer policy $T_t^\theta \geq 0$. With $\mu \geq \bar{\mu}$, a transfer policy is optimal only if (41) and (42) are satisfied. (46) together with (48) and (49) imply that $\phi_t = w(\mu) = u'\left(\mu f\left(q^0\right) + (1-\mu) f\left(q^{\bar{\theta}}\right)\right)^{-1}$, for all t. Hence, it has to be that T_t^θ satisfies

$$V_\theta'(T_t^\theta) = \phi = w(\mu)$$

Therefore $T_t^\theta = T^\theta$ is constant, and market clearing requires $T^\theta = \bar{\theta} q^{\bar{\theta}}$. Hence, dirty firms should not conduct R&D whenever $\mu \geq \bar{\mu}$, and the transfer should implement $i(\bar{\theta}) = 0$. That is, it should be that (we set $T^0 = 0$)

$$V_0\left(0\right) - V_\theta(T^\theta) < \frac{\gamma w}{\beta s}$$

where we can easily compute $V_0(0) - V_\theta(T^\theta)$ to be

$$V_0(0) - V_\theta(T^\theta) = \frac{f\left(q^0\right) - wq^0}{1-\beta} - \frac{f\left(q^{\bar{\theta}}\right) - wq^{\bar{\theta}} - w\bar{\theta}q^{\bar{\theta}} + wT^\theta}{1-\beta}$$

Using the market clearing condition in the market for permits, we obtain that $i(\bar{\theta}) = 0$ if

$$f\left(q^0\right) - wq^0 - \left[f\left(q^{\bar{\theta}}\right) - wq^{\bar{\theta}}\right] < (1-\beta)\frac{\gamma w}{\beta s}$$

Since $\mu \geq \bar{\mu}$ and $F'' < 0$, this condition is satisfied since the LHS is less than $wF'(\bar{\mu})$ which is equal to the RHS by (45).

(ii) Case when $\mu < \bar{\mu}$ but close to $\bar{\mu}$

In this case, the efficient allocation has some dirty firms investing in R&D according to (44). Therefore, it must be that (47) holds with equality, or,

$$\left[-\phi m_+^0 + \beta V_0\left(m_+^0 + T_+^0\right)\right] - \left[-\phi m_+^\theta + \beta V_\theta(m_+^\theta + T_+^\theta)\right] = \frac{w\gamma}{s} \tag{50}$$

We can then write $V_\theta(m)$ as

$$\begin{aligned} V_\theta\left(m\right) &= \max_{q,i,m_+^0,m_+^\theta} f\left(q\right) - w\left(q + \gamma i\right) + \phi m - \phi\theta q \\ &\quad + is\frac{w\gamma}{s} - \phi m_+^0 + \beta V_0\left(m_+^0 + T_+^0\right) - \frac{w\gamma}{s} \\ s.t.\ 0 &\leq i \leq 1 \end{aligned}$$

or, using the solution for q^0 for clean firms,

$$\begin{aligned} V_\theta\left(m\right) &= \max_{q,i,m_+^0,m_+^\theta} f\left(q\right) - wq + \phi m - \phi\theta q \\ &\quad - f(q^0) - \phi m^0 + wq^0 + V_0(m^0) - \frac{w\gamma}{s} \end{aligned}$$

We need to check whether we can obtain (50) using this formulation. Let q^θ be the solution to the dirty firm's problem given wage w. Then

$$V_\theta(m) - V_0(m^0) = f(q^\theta) - f(q^0) - wq^\theta - \phi\theta q^\theta + wq^0 + \phi(m - m^0) - \frac{w\gamma}{s}$$

Using this expression into (50) we obtain that $i(\bar{\theta}) \in (0,1)$ only if

$$\frac{w\gamma}{s} = -w\left(m_+^0 - m_+^\theta\right) + \beta\left[f(q_+^0) - f(q_+^\theta) - w_+ q_+^0 + w_+(1+\theta)q_+^\theta + w_+(T_+^0 - T_+^\theta) + \frac{w_+\gamma}{s}\right]$$

where we have used that $\phi_+ = w_+$, as this is a necessary condition for efficiency. Using $F'(\mu_+)$, we can rewrite the above equation as

$$-\frac{w}{w_+}s\left(m_+^0 - m_+^\theta\right) + \beta\left[sF'(\mu_+) + s(T_+^0 - T_+^\theta) + \gamma\right] = \frac{w}{w_+}\gamma$$

Comparing this equation with (44) the efficient outcome with $i = i^*$ given by (44) is implemented only if

$$-\frac{w}{w_+}s\left(m_+^0 - m_+^\theta\right) + \beta s(T_+^0 - T_+^\theta) + \beta\gamma - \frac{w}{w_+}\gamma = -\gamma + \beta\gamma$$

or

$$\beta s(T_+^0 - T_+^\theta) = \left(\frac{w}{w_+} - 1\right)\gamma + \frac{w}{w_+}s\left(m_+^0 - m_+^\theta\right) \tag{51}$$

Since $\phi_+ = w_+$, the consumers' first order condition gives $u'^{-1} = w(\mu)$. Thus,

$$\frac{\beta\phi_+}{\phi} = \beta\frac{u'(c(\mu))}{u'(c(\mu_+))} < 1$$

where the last inequality follows from the fact that we assume that μ is close to $\bar{\mu}$. In this case, the efficient allocation implies that the investment in R&D decreases so that μ can be close to μ_+, so that the inequality holds. In that case, $m_+^0 = m_+^\theta = 0$, so that (51) gives

$$T^\theta(\mu_+) = \left(1 - \frac{u'(c(\mu_+))}{u'(c(\mu))}\right)\frac{\gamma}{\beta s} + T^0(\mu_+)$$

Market clearing requires that

$$(1-\mu)T^\theta(\mu) + \mu T^0(\mu) = (1-\mu)\theta q^\theta$$

Therefore,

$$\begin{aligned} T^0(\mu_+) &= (1-\mu_+)\theta q_+^\theta - (1-\mu_+)\left(1 - \frac{u'(c(\mu_+))}{u'(c(\mu))}\right)\frac{\gamma}{\beta s} \\ T^\theta(\mu_+) &= (1-\mu_+)\theta q_+^\theta + \mu_+\left(1 - \frac{u'(c(\mu_+))}{u'(c(\mu))}\right)\frac{\gamma}{\beta s} \end{aligned}$$

Notice that if μ_+ is close enough to μ (which will be the case when $i(\theta)$ is sufficiently close to zero), then $T^0(\mu) > 0$, so that the optimal policy is to grant some permits to clean firms. As $T^\theta(\mu)$ is not sufficient for dirty firms to pledge the required permits, they will have to purchase the missing permits from clean firms, thus, effectively subsidizing them. This subsidy makes being "clean" more attractive and incentivizes investment in R&D. Note that this is in addition to having to give up revenue from permits. This additional incentive is necessary since efficiency requires that $w = \phi$, so that the price of permits is pinned down by the wage *and* the wage is pinned down by the marginal utility of consumption.

(iii) Case when μ is far lower than $\bar{\mu}$

Finally, we consider the case where μ is far lower than $\bar{\mu}$, so that $u'^{*}(c(\mu_{+}))/u'^{*}(c(\mu)) < \beta$. Then emissions trading cannot implement the efficient allocation. Indeed, optimality requires that $w(\mu) = \phi(\mu)$. But this would imply that $\phi(\mu) < \beta\phi(\mu_{+})$. This is not consistent with an equilibrium, as it implies an excess demand of permits by firms who will want to resell them in the next period.

In summary, when the measure of dirty firms is greater than a critical threshold, the efficient allocation is not implementable via the use of an emissions trading system. Equilibrium under emissions trading either makes technology adoption by dirty firms too slow, or it distorts production of dirty firms relative to the first best. Below we show that fiscal policies do not seem to suffer from this drawback. As in the case without endogenous technology change, a tax scheme can implement the first best.

4.2 Taxes

We denote the value of a *dirty* firm entering the futures market by $V_{\bar{\theta}}$, and the value for a clean firm by V_0. Hence, for $\theta \in \{0, \bar{\theta}\}$, V_θ is defined by

$$\begin{aligned} V_\theta(\mu) &= \max_{q,i} f\left(q\right) - w\left(q + \gamma i\right) - \tau\left(\theta q | \mu\right) \\ &\quad + is\beta V_0(\mu_+) + \left[i(1-s) + (1-i)\right]\beta V_\theta(\mu_+) \\ s.t.\ 0 &\leq i \leq 1 \end{aligned}$$

The first order conditions are

$$\begin{aligned} f'^{\theta}) - w - \theta\tau'(\theta q|\mu) &= 0 & (52) \\ -w\gamma + s\beta V_0\left(\mu_+\right) - s\beta V_\theta(\mu_+) &\leq 0 (= 0, \text{ if } i > 0, > 0, \text{ if } i = 1) & (53) \end{aligned}$$

Clearly, optimality requires that

$$\tau'(\theta q|\mu) = w(\mu)$$

so that the optimal tax is linear in the quantity of emissions; i.e., $\tau(\theta q|\mu) = w(\mu)\theta q + x^\theta(\mu)$. To induce investment, the tax must be such that (53) holds with equality whenever $i^* > 0$. Using q^0 as the optimal choice of clean firms, and (53) at equality, we can rewrite $V_{\bar{\theta}}(\mu)$ as

$$V_{\bar{\theta}}(\mu) = \max_q f\left(q\right) - wq - \tau\left(\theta q|\mu\right) - \left[f(q^0) - wq^0 - \tau(0|\mu)\right] + V_0(\mu) - w\gamma/s$$

Therefore,

$$V_{\bar{\theta}}(\mu_+) - V_0(\mu_+) = f\left(q_+^\theta\right) - w(\mu_+)q_+^\theta - \tau\left(\theta q_+^\theta|\mu_+\right) - \left[f(q_+^0) - w(\mu_+)q_+^0 - \tau(0|\mu_+)\right] - w(\mu_+)\gamma/s$$

and using this expression back in (53), we obtain that $i^* \in (0,1)$ only if

$$\begin{aligned} V_0(\mu_+) - V_\theta(\mu_+) &= \frac{w(\mu)\gamma}{s\beta} \\ \left[f(q_+^0) - w(\mu_+)q_+^0 - x^0(\mu_+)\right] - \left[f\left(q_+^\theta\right) - w(\mu_+)q_+^\theta - w(\mu_+)\theta q_+^\theta - x^\theta(\mu_+)\right] + \frac{w(\mu_+)\gamma}{s} &= \frac{w(\mu)\gamma}{s\beta} \\ s\beta F'(\mu_+) + \beta\gamma - s\beta\frac{x^0(\mu_+) - x^\theta(\mu_+)}{w(\mu_+)} &= \frac{w(\mu)\gamma}{w(\mu_+)} \end{aligned}$$

Comparing this last expression with (44), we obtain that the tax policy can implement i^* if

$$s\beta\frac{x^0(\mu_+) - x^\theta(\mu_+)}{w(\mu_+)} = \left[1 - \frac{w(\mu)}{w(\mu_+)}\right]\gamma$$

or, using the consumers' first order condition, if

$$x^0(\mu_+) - x^\theta(\mu_+) = \frac{1}{u'(c(\mu_+))}\left[1 - \frac{u'(c(\mu_+))}{u'(c(\mu))}\right]\frac{\gamma}{s\beta}$$

In particular, if $x^0 = 0$ then $x^\theta(\mu) < 0$ and dirty firms should receive a corresponding lump-sum subsidy. Again, the intuition is that without this subsidy, the value of being dirty would be too low and dirty firms would invest too much in R&D relative to the first best when the marginal tax rate is $w(\mu)$.

Thus, a tax scheme is less constrained in achieving the optimum than an emissions trading system. Equilibrium under cap-and-trade imposes the additional condition that $\phi = w$, which reduces the range of policies available and, as a result, may fail to attain the first best. Modeling explicitly the money-like feature of permits implies that there are additional requirements that need to be satisfied in order for permits to be valued in equilibrium. These requirements are binding, in the sense that they restrict the set of environments in which cap-and-trade can be as effective as a tax.

5 Conclusion

Emissions trading mechanisms have been proposed, and in some cases implemented, as a tool to reduce pollutants. We used insights from dynamic mechanism design in monetary economics to derive properties of *optimal dynamic emissions trading mechanisms*. We demonstrated that a state-contingent tax system can do at least as well as a cap-and-trade system in most cases, and there is a sense in which it can dominate when there is endogenous progress in clean technologies. More generally, we argue that policy-makers should think about permit-issue in a manner similar to that used by central banks. The optimal policy must ensure that the price of emissions increases over time. In the absence of aggregate risk, there is no role for banking, and the optimum can be supported even if the permits expire at the end of the specified period of time. In the presence of aggregate risk, the optimal supply of permits is not constant over time and must respond to the shocks affecting the economy. Finally, when firms can choose the level of technological progress in green technologies, emissions trading cannot implement the efficient allocation if there are a many "dirty firms." The reason is that emissions trading either makes technology adoption by such firms too slow, or it must distort production relative to the first best. We showed that fiscal policies do not suffer from this drawback.

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6 An Extension: Futures Market

In the emissions trading system studied in the body of the paper we assumed that the issuing authority assigns permits to firms at the start of a new remittance period. In this section, we show how our model can be extended to study the market for permits when the government sells permits rather than transferring them lump-sum and free of charge.[12]

Assume that firms receive signal $s = \theta + \varepsilon$ on the realization of their shock, θ, at the start of the market. The random term ε is drawn from a distribution F and $E(\varepsilon_i) = 0$, for all i. Given this structure, the firm's signal is also a firm's best guess for the true value of θ. Once s is observed, a firm can access a futures market to acquire or sell permits at a price p, for delivery at the remittance date. At this stage, the government sells an amount T of permits (buys if $T < 0$).

Then the true shock is realized and firms decide on their production and emission levels. At the remittance date, a spot market for permits opens, where firms can trade their permits at a price ϕ. Each firm then presents an amount of permits equal to the amount of emissions y.

We denote the value of entering the futures market with m permits and shock s by $V(m; s)$. We denote the value of entering the spot market for permits with m permits and shock value s as $W(m; \theta)$. Then, $V(m; s)$ is defined by

$$\begin{aligned} V(m; s) &= \max_x E_{\theta|s} W(m - x, x; \theta) \\ s.t.\ x &\leq m \end{aligned} \tag{54}$$

while $W(m)$ solves

$$W(m, x; \theta) = \max_{x,q,y,m_+} f(q) - wq - h(\theta q - y) + \phi(m - y) + px + \tau - \phi m_+ + \beta E_s V(m_+; s) \tag{55}$$

$$s.t.\ \ 0 \leq y \leq \theta q \tag{56}$$

where τ is a lump-sum transfer. Using (55) to replace W in (54), we obtain

$$\begin{aligned} V(m; s) &= \max_{x \leq m} px + \int_{\theta|s} \left[\max_{q,y} f(q) - wq - h(\theta q - y) + \phi(m - x - y) \right] \\ &\quad + \max_{m_+} \beta E_s V(m_+; s) - \phi m_+ \\ &\quad s.t.\ \ 0 \leq y \leq \theta q \end{aligned}$$

Given M, the market clearing conditions are

$$\int x(s)\, dH(s) + T = 0 \tag{57}$$

$$\int y(\theta, s)\, dH(s)\, dG(\theta) + m_+ = M + T \tag{58}$$

$$\int f(q(\theta)) - h(\theta q(\theta) - y)\, dG(\theta) = c \tag{59}$$

[12] More generally, we could investigate competing mechanisms for allocating permits in environments that include frictions, as in Eeckhout and Kircher (2010). This, however, is beyond the scope of the present paper.

The stock of permits follows the law of motion

$$M_+ = M - \int y(\theta)\, dG(\theta) + T$$

Given a policy $\{T_t\}$, an equilibrium is a list of quantities and emissions $\{c_t, q_t(\theta), y_t(\theta)\}$, permit-trading decisions $\{x_t(\theta), m_t(\theta)\}$, and prices $\{p_t, \phi_t\}$, such that, given prices, the list of decision variables solves the firms' and consumers' problems and markets clear. An equilibrium is stationary whenever the list of quantities and emissions is time independent; i.e., when $\{c_t, q_t(\theta), y_t(\theta)\} = \{c, q(\theta), y(\theta)\}$, for all t.

We demonstrate that for any stationary policy T, there is a unique stationary equilibrium. We first solve the firm's problem. The first order conditions give

$$\begin{aligned} f'(q) - \theta h'(\theta q - y) &= w - \theta\phi\lambda_\theta(\theta) && (60)\\ h'(\theta q - y) - \phi - \phi\lambda_\theta(\theta) + \phi\lambda_0(\theta) &= 0 && (61)\\ p - \phi\lambda(s) - \phi &= 0 && (62)\\ \beta E_s V_{m_+}(m_+; s) &\leq \phi = \text{if } m' > 0 && (63)\end{aligned}$$

where $\phi\lambda(s)$ is the Lagrange multiplier on the firm's constraint in the futures market, and $\phi\lambda_\theta(\theta)$, $\phi\lambda_0(\theta)$ are the multipliers on the constraints related to emissions reduction. Expression (62) already incorporates the fact that ϕ will not depend on idiosyncratic shocks. Notice from (63) that all firms will exit the market for permits holding the same amount of permits for the next period. The envelope condition gives

$$V_m(m; s) = \phi(1 + \lambda(s)) \tag{64}$$

The workers' decision is still given by (2) and, using market clearing, we obtain an expression for the wage

$$wu'\left(\int f(q(\theta)) - h(\theta q(\theta) - y(\theta))\, dG(\theta)\right) = 1 \tag{65}$$

From (62), it is clear that either $\lambda(s) > 0$, for all s, and $p > \phi$, or $\lambda(s) = 0$, for all s, and $p = \phi$. If $p > \phi$, then all firms sell their permits, so that $T = -M < 0$. In addition, (58) implies that $\int y(\theta, s) + m_+ = 0$. Since $y(\theta, s) \geq 0$ and $m_+ \geq 0$, this implies that $y(\theta, s) = 0$, for all s, θ. Clearly this is not the efficient equilibrium. So, the only candidate efficient equilibrium is one where $\lambda(s) = 0$, for all s, so that $p = \phi$. This is equivalent to an equilibrium where the issuing authority would buy or sell permits in the spot market during the remittance period. Given $p = \phi$, the equilibrium is as in the text, and we can set $x(s) = -T$ and $y(\theta, s) = y(\theta, s')$, for all (s, s'), since firms are indifferent between holding permits across the two markets.